Three mathematical models of photons

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Motivation

There is a growing interest in the interaction of matter and light, as well as the single quantum technologies where the main element is a photon. But there is not a complete understanding of what a photon is exactly. We compare 3 different mathematical models of a photon.

The first model: Maxwell equations and their quantization

The theory of photons is based on the quantization of Maxwell equations which was reduced to the quantization of an infinite number of linear harmonic oscillators.

Maxwell equation for a vacuum

$$\triangle \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0, \qquad \mathrm{div}\,\boldsymbol{A} = 0.$$

The normalized wave function for the energy operator:

$$\Psi_{kn} = \frac{1}{\sqrt{\sqrt{\pi}2^n n!}} H_n(\xi_k)\, e^{-\xi_k^2/2}.$$

The vector potential is an operator and for one mode:

$$\hat{A}_k = \sqrt{\frac{2\pi\hbar c}{kV}}\left(a_{k\alpha}e^{i(\boldsymbol{kr}-\omega t)} + a^+_{k\alpha}e^{-i(\boldsymbol{kr}-\omega t)}\right),$$

The quantization procedure provides the correct energy and momentum of the photon and perfectly describes the creation and annihilation of photons. The quantization of the field is the cornerstone of the quantum field theory and, especially, quantum electrodynamics. Despite the great success of the quantization of light, the physics of this procedure is not so clear: what is there that oscillates and where are photons located in time and space? The definition of a photon as a first excited state of a single mode of the quantized electromagnetic field is rather abstract.

The second model: Photon-soliton models

If we look at the photon as a stable object that is localized in space, the same definition is valid for a soliton. In these models [1-3] the non-linearity in the Maxwell equations is introduced by small, finite components of polarization and magnetization in a vacuum along the direction of propagation of light.

The proposed equation of the vector potential for the dimensionless funct $A_k = \sqrt{\hbar c k} F$:

$$\frac{1}{k^2\rho_k}\frac{\partial F}{\partial \rho_k} + \mu F + \mu\left[(\eta_k - 2)\frac{\partial}{\partial \eta_k} - i\eta_k - i\frac{\partial^2}{\partial \eta_k^2} - 2i|F|^2\right]F = 0,$$

With the one-soliton solution:

$$F = b\,\mathrm{sech}(b\eta_k)\exp\left[i\left(\eta_k + \frac{\mu k^2\rho_k^2}{2} - \frac{b^2}{2} + \gamma\right)\right], \qquad b = m\exp\left(-\frac{\mu k^2 \rho_k^2}{2}\right)$$

where μ is a dimensionless parameter, m is constant, γ is the phase and $\eta_k = \omega t - \boldsymbol{kr}$ and the polar radius $k\rho_k$ are the variables.

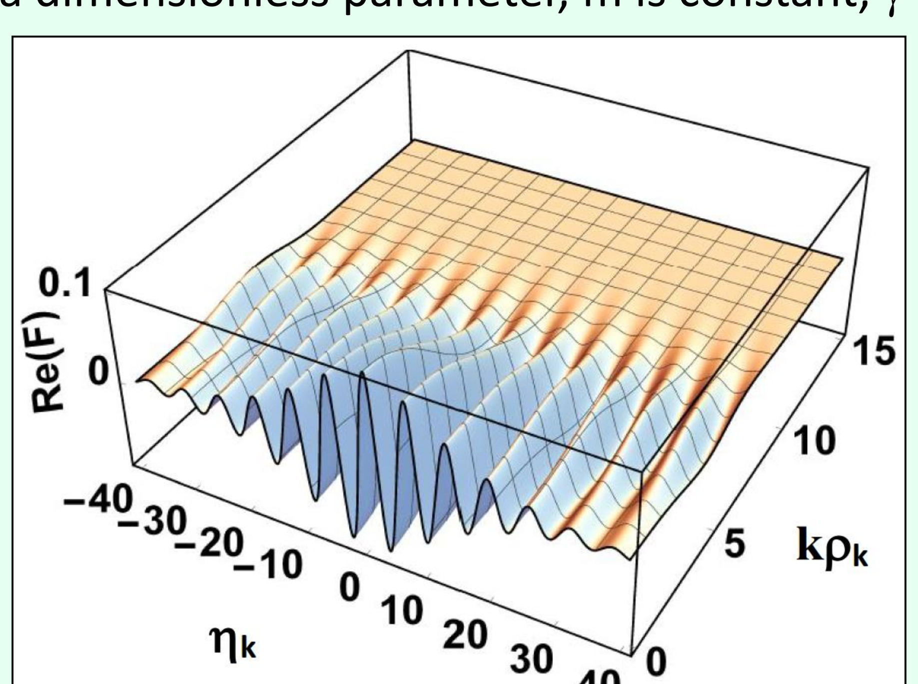

FIG. One-soliton solution with the parameter μ = m = 0.1.

The third model: the new model

Combining the mathematics of the first and second models, a new photon model is proposed [4]. The free propagating photons are described by the vector potential as a product of two functions G(ηk) and R(ρk), $A_{\vec{k}} = NR(\rho_{\vec{k}})G(\eta_{\vec{k}})$. The function, which is a product of harmonic oscillator eigenfunction with the coordinate $\eta_k = \omega t - \boldsymbol{kr}$ and the Gaussian functions of the transverse coordinates $\tau_k = k^2\rho_k^2/2 = k^2(x^2+y^2)/2$.

$$\left[\frac{d^2}{d\eta_{\vec{k}}^2} - s^4\eta_{\vec{k}}^2 + \lambda\right]G(\eta_{\vec{k}}) = 0, \qquad \frac{dR(\rho_{\vec{k}})}{d\tau_{\vec{k}}} + \mu R(\rho_{\vec{k}}) = 0$$

As a result we get the function that describes the free propagation of one-mode n photons with two dimensionless parameters μ and s:

$$A_{\vec{k}n} = k\sqrt{\frac{\hbar c\mu}{s\sqrt{\pi}2^{n-2}n!}}H_n(s\eta_{\vec{k}})\exp\left(-\frac{s^2\eta_{\vec{k}}^2}{2} - \mu\tau_{\vec{k}}\right).$$

The interaction potential between the photons and the charged particle differs from the potential derived by the traditional quantization method only with the definition of the harmonic oscillator coordinates.

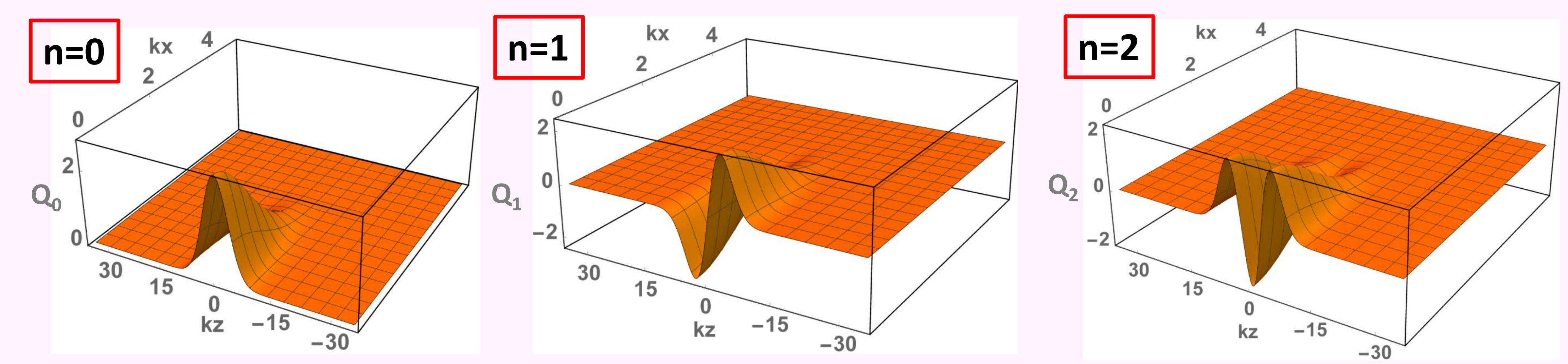

FIG. The cross-section at y=0 of the dimensionless functions $Q = A_{\vec{k}n}/k\sqrt{\hbar c}$ of the vacuum, one- and two-photons state for parameters s = 0.3, μ = 1 and t = 0 are depicted.

The reflection and refraction of photons on the boundary between two dielectrics, with the refractive indexes n_1 and n_2, is considered [4]. The amplitudes of the reflected and transmitted photons are determined by the Fresnel formulae, such as for the plane waves, but the transverse size μ_x^t of the transmitted photons in the plane of incidence changes with the angle of incidence θ_i changing:

$$\mu_x^t(\theta_i) = \mu_x^i\,\frac{1-\sin^2\theta_i}{1-\frac{n_1^2}{n_2^2}\sin^2\theta_i}$$

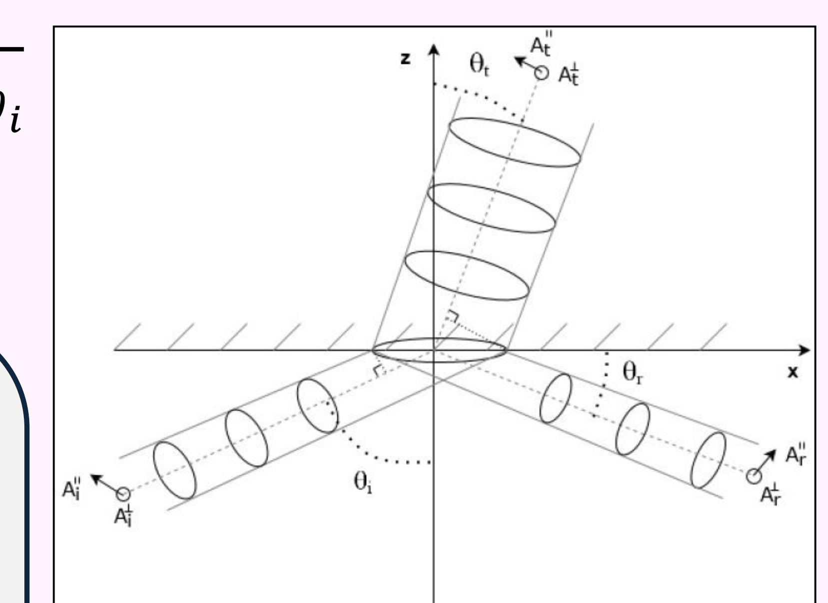

FIG. The geometry of incidental, reflected and refracted photons on the surface.

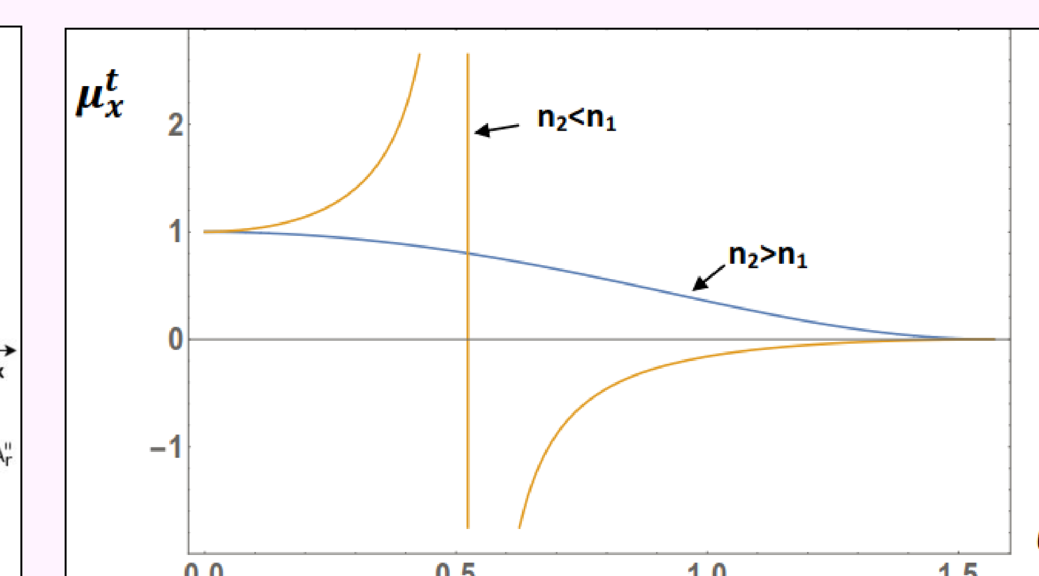

FIG. The transverse size of the transmitted photons depends on the angle of incidence for a different relation of the refractive index.

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