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The new mathematical model of photons

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Motivation

- There is a growing interest on the interaction of matter and light, as well as the single quantum technologies where the main element is a photon. But there is not a complete understanding of what a photon is exactly.
- **Traditional approach:** the quantization procedure for the Maxwell equations which is reduced to the quantization of an infinite number of linear harmonic oscillators. The quantization procedure provides the correct energy and momentum of the photon and perfectly describes the creation and annihilation of photons.

➢ The quantization of the field is the cornerstone of the quantum field theory and, especially, quantum electrodynamics.

➢ Despite the great success of the quantization of light, the physics of this procedure is not so clear: what is there that oscillates and where are photons located in time and space?

➢ The definition of a photon as a first excited state of a single mode of the quantized electromagnetic field is rather abstract.

The photon-soliton model

- To get the soliton type equation, we use the Maxwell equations introducing the small, finite components of polarization and magnetization in a vacuum along the direction of the propagation of light making it nonlinear [1-3].
- The proposed equation of the vector potential for the dimensionless function $A_k = \sqrt{\hbar c k} F$

$$\frac{1}{k^2 \rho_k} \frac{\partial F}{\partial \rho_k} + \mu F + \mu \left[(\eta_k - 2) \frac{\partial}{\partial \eta_k} - i \eta_k - i \frac{\partial^2}{\partial \eta_k^2} - 2i |F|^2 \right] F = 0,$$

- with one-soliton solution:

$$F = b \,\mathrm{sech}(b \eta_k) \exp \left[i \left(\eta_k + \frac{\mu k^2 \rho_k^2}{2} - \frac{b^2}{2} + \gamma \right) \right],$$

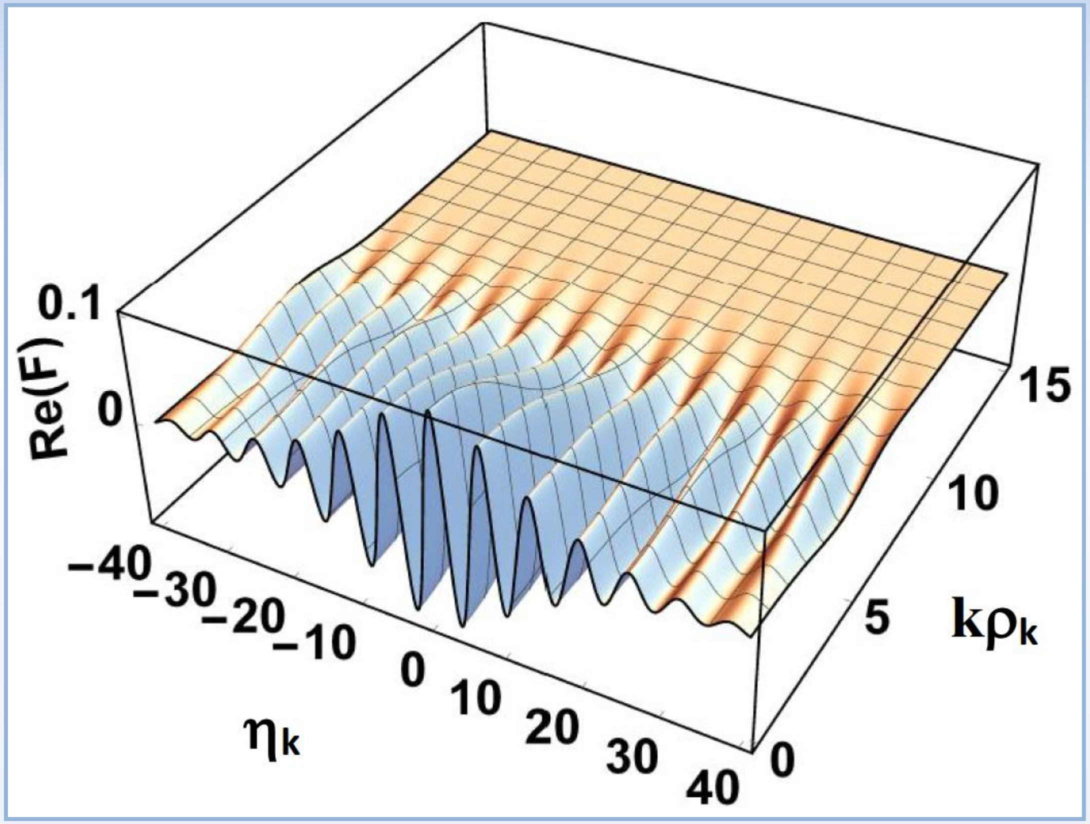
$$b = m \exp \left(- \frac{\mu k^2 \rho_k^2}{2} \right)$$

- where μ is a dimensionless parameter, m is constant, γ is the phase and $\eta_k = \omega t - \mathbf{kr}$ and the polar radius $\rho_k = (x^2 + y^2)^{1/2}$ are variables.

➢ In this model, the solitons are identified with photons and the localization of them in space is well defined.

The photon-soliton model

- The one-soliton solution with the parameter μ = m = 0.1.



➢ The applicability of this model for photons is an open question. The vector potential of this model looks physically suitable, but until now there has not been offered any proposal about how to check the conformity of the model to photons. So, it can serve as an interesting model of compact 3D electromagnetic solitons.

The proposed new mathematical model of photons

- Combining the mathematics of the first and second models, the new photon model is proposed [4]. The free propagating photons are described by the vector potential as a product of two functions G(ηk) and R(ρk), $A_{\vec{k}} = NR(\rho_{\vec{k}})G(\eta_{\vec{k}})$. The function, which is a product of harmonic oscillator eigenfunction with the coordinate $\eta_k=\omega t-\boldsymbol{kr}$ and the Gaussian functions of the transverse coordinates $\tau_k=k^2\rho_k^2/2=k^2(x^2+y^2)/2$

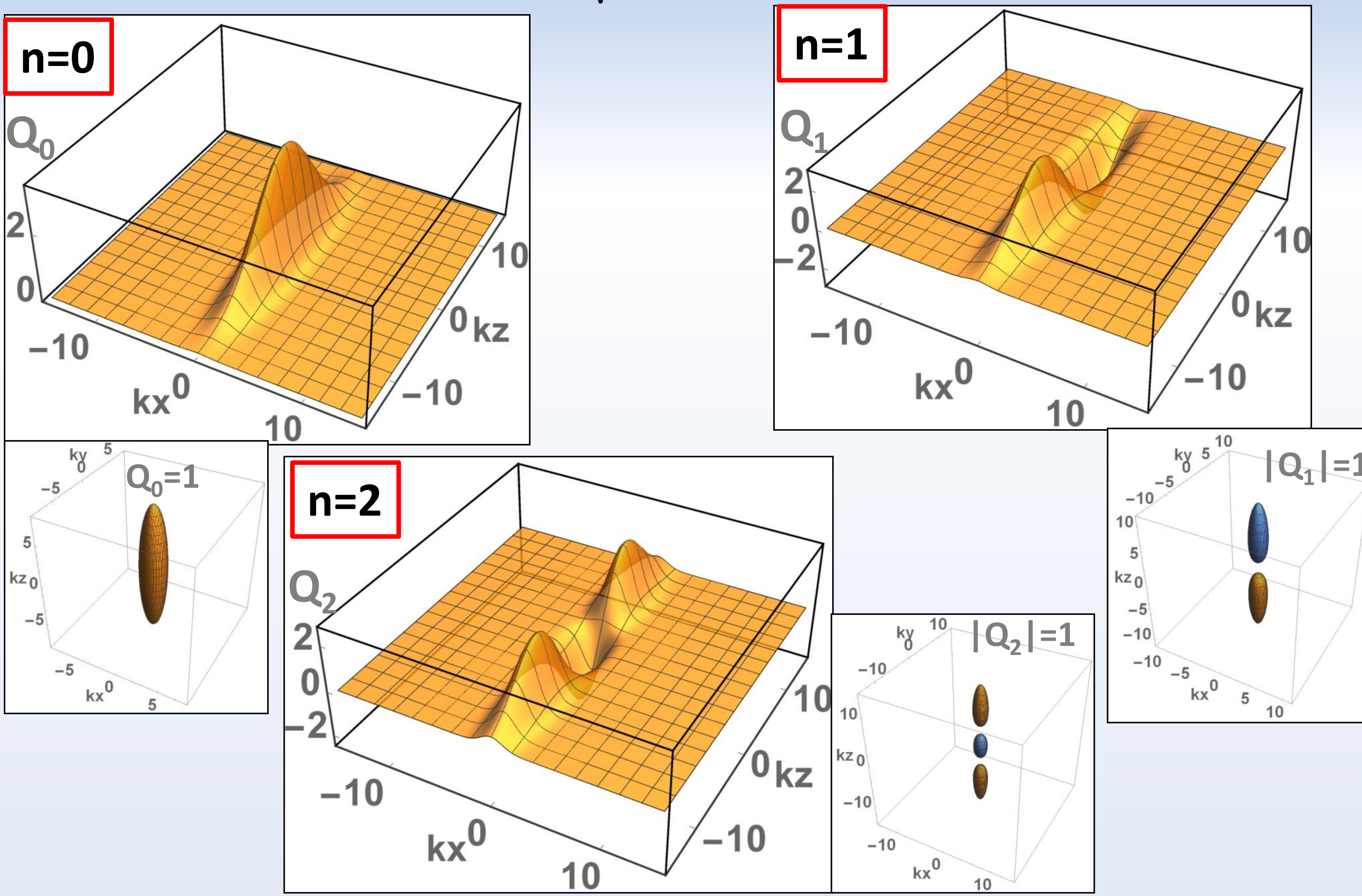
$$\left[\frac{d^2}{d\eta_{\vec{k}}^2} - s^4\eta_{\vec{k}}^2 + \lambda\right]G(\eta_{\vec{k}}) = 0, \qquad \frac{dR(\rho_{\vec{k}})}{d\tau_{\vec{k}}} + \mu R(\rho_{\vec{k}}) = 0$$

- As result, we get the function that describes the free propagation of one-mode n photons with two dimensionless parameters s and μ, which determine the longitudinal and transverse size of photons

$$A_{\vec{k}n} = k\sqrt{\frac{\hbar c\mu}{s\sqrt{\pi}2^{n-2}n!}}H_n(s\eta_{\vec{k}})\exp\left(-\frac{s^2\eta_{\vec{k}}^2}{2} - \mu\tau_{\vec{k}}\right).$$

- The interaction potential between the photons and the charged particle differs from the potential derived by the traditional quantization method only with the definition of the harmonic oscillator coordinates.

The cross-section of the dimensionless functions $Q = A_{\vec{k}n}/k\sqrt{\hbar c}$ of the vacuum, one- and two- photons state for parameters s = 0.2, μ = 1 and t = 0



Conclusions

- In the proposed new model, we get the same vector potential as in the traditional quantization method (which contains the annihilation and creation operators of photons and the coordinates of the charged particle on which the potential acts). The difference is that in our model we know the physical meaning of the coordinate. Additionally, we add the vector potential that depends on the transversal coordinates. Therefore in our model the photon is the three dimensional object.
- The model contains two unknown dimensionless parameters s and μ , which determine the longitudinal and transverse size of photons.
- Of course, the proposed model is only a model, and it should be tested in different processes where photons participate, for example, the reflection and refraction of photons on the interface of the two dielectrics.

References

[1] I. Bersons, Latv. J. Phys. Tech. Sci. 50, 2, 60 (2013).
[2] I. Bersons, R. Veilande, and A. Pirktinsh, Phys. Scr. 89, 045102 (2014);
[3] I. Bersons, R. Veilande, and O. Balcers, Phys. Scr. 91, 065201 (2016); Phys. Scr. 95, 025203 (2020).
[4] I. Bersons, R. Veilande, and O. Balcers *«Mathematical models of photons»,* submitted to the *Journal of Mathematical Physics.*