



DISPERSION ENGINEERING OF WHISPERING GALLERY MODE RESONATORS FOR FREQUENCY COMB GENERATION AND TELECOMMUNICATION APPLICATIONS

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EIROPAS SAVIENĪBA

Eiropas Reģionālās
attīstības fonds

IEGULDĪJUMS TAVĀ NĀKOTNĒ



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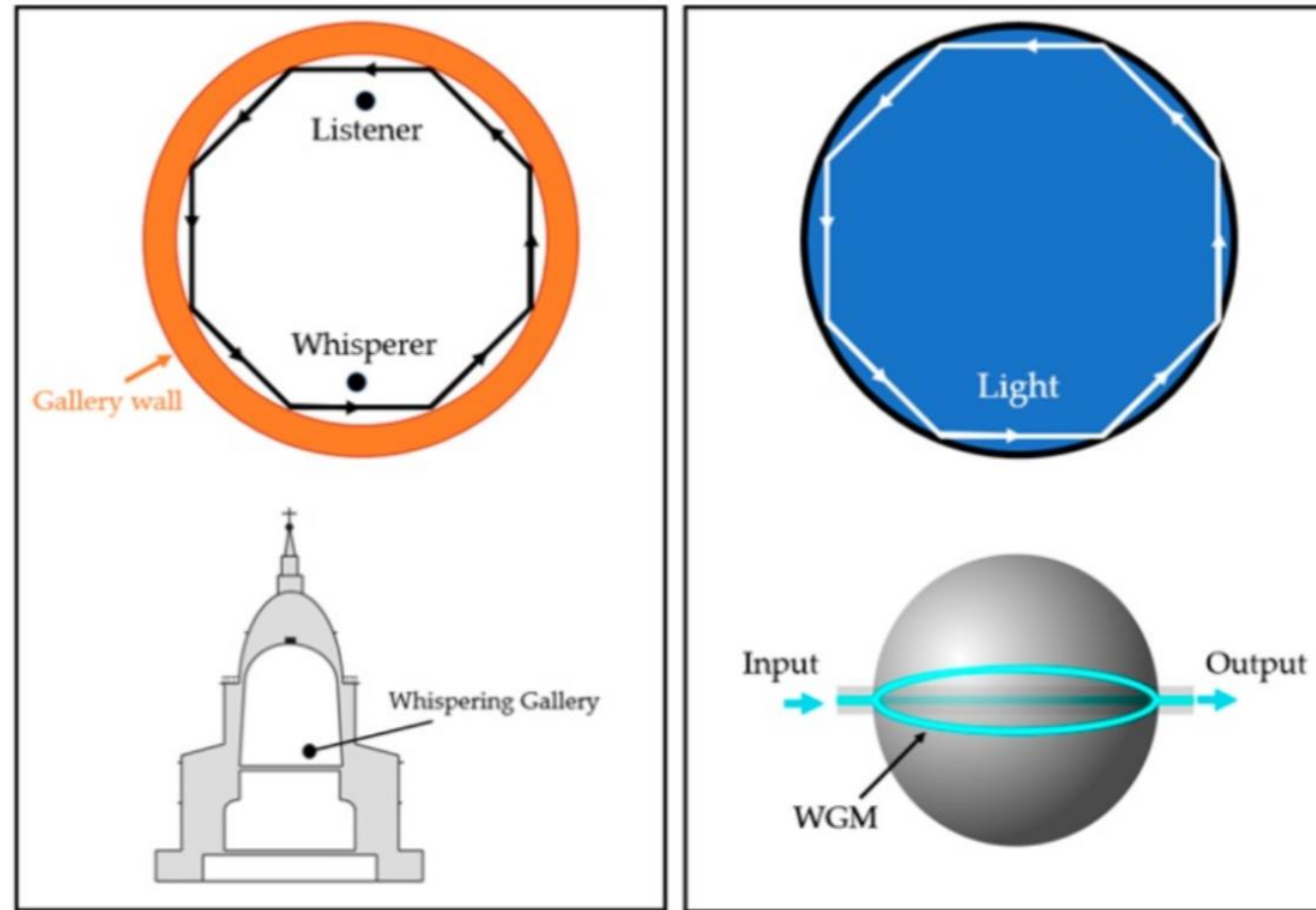
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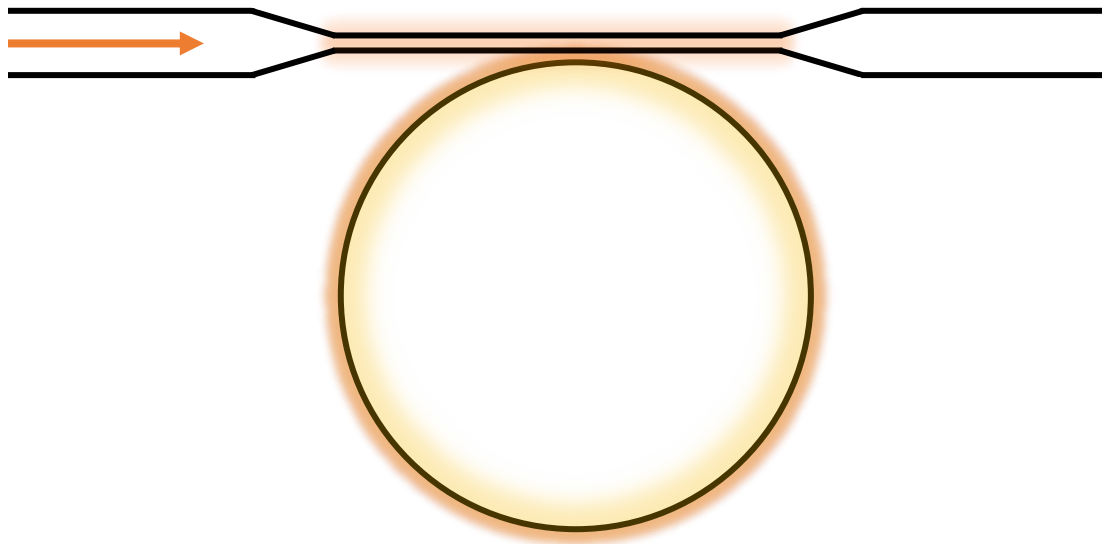
Whispering Gallery in St Paul's Cathedral [1,2]



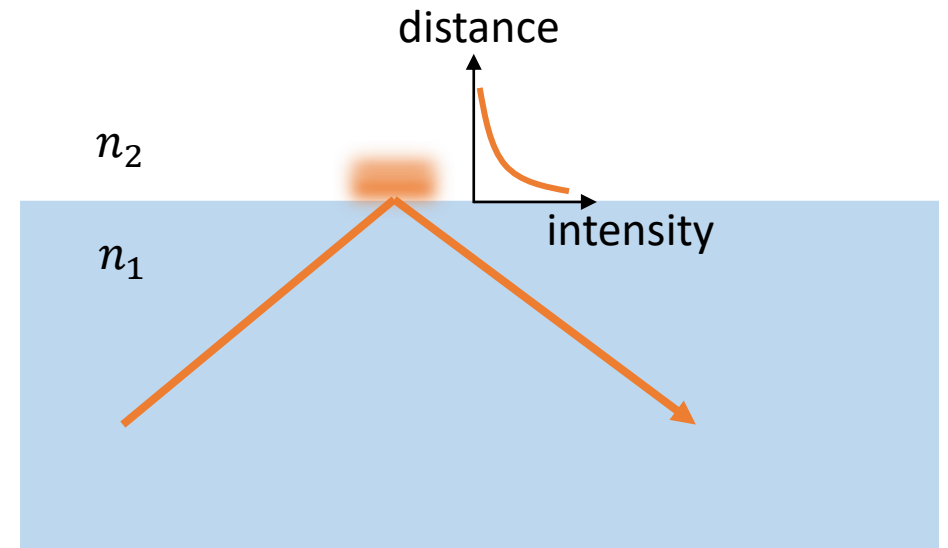
Whispering Gallery Mode Resonators (WGMRs) [3]



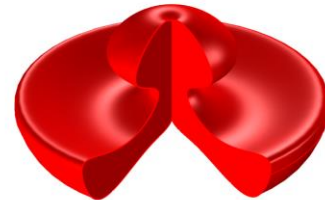
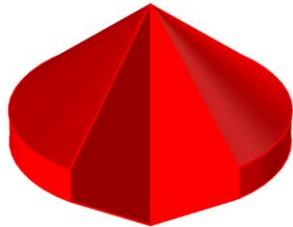
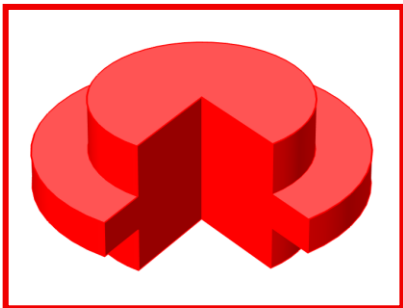
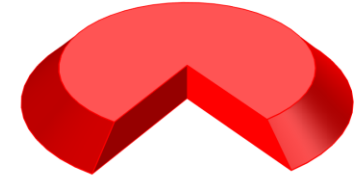
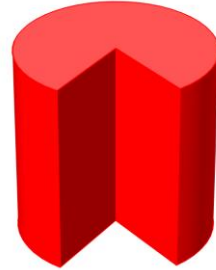
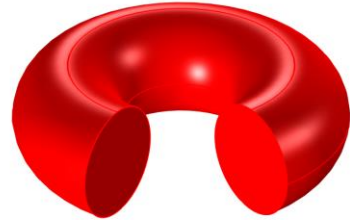
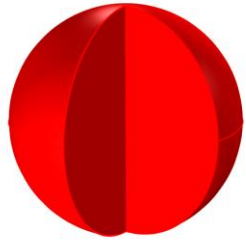
Coupling WGMR with light using evanescent field



$$2\pi Rn = m\lambda$$



Different WGMR types



$$2\pi Rn = m\lambda$$

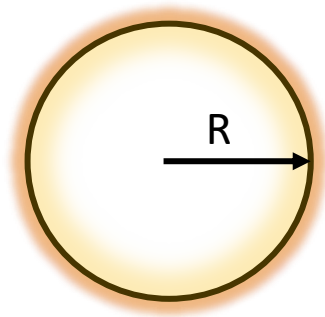
Free Spectral Range (FSR), applications in telecommunications

$$2\pi Rn = m\lambda$$

$$\lambda = \frac{c}{\nu}$$

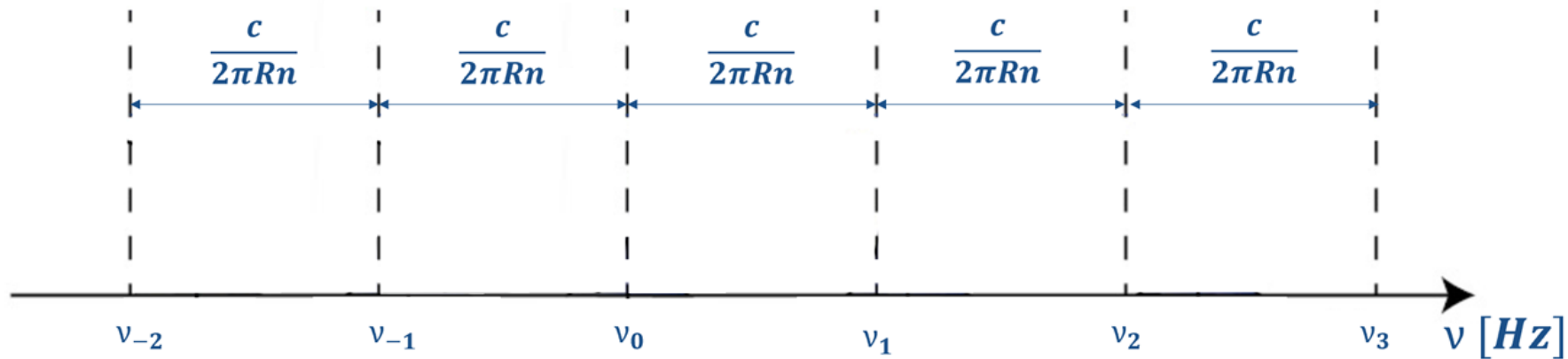
$$\nu_1 = \frac{mc}{2\pi Rn}$$

$$\nu_2 = \frac{(m+1)c}{2\pi Rn}$$

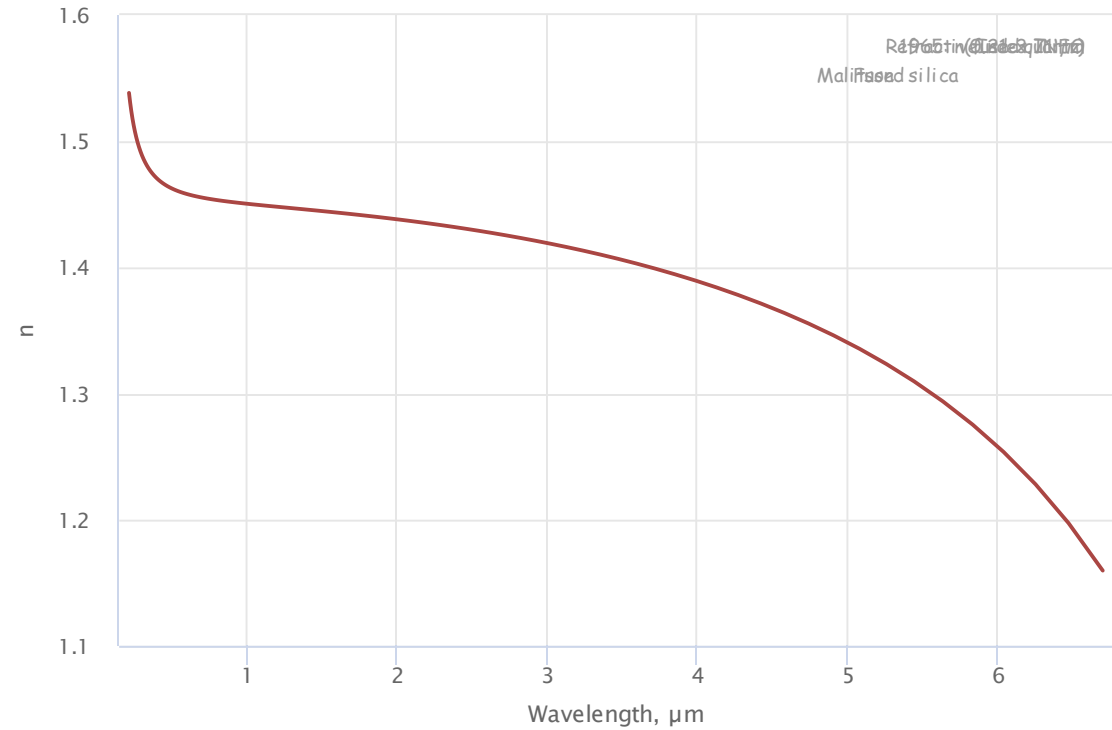
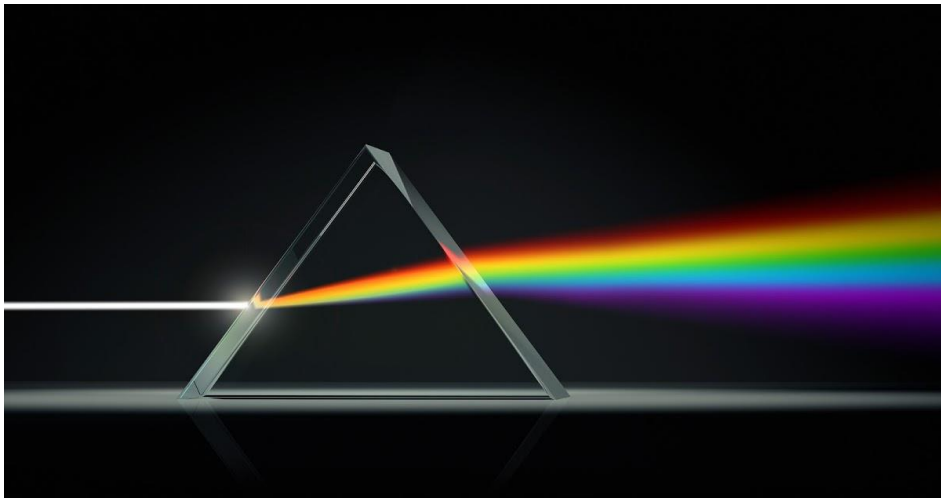


$$\Delta\nu = \nu_2 - \nu_1 = \frac{c}{2\pi Rn} = \text{FSR}$$

$$R_{(\text{FSR}=100\text{GHz})} = 332 \mu\text{m}$$

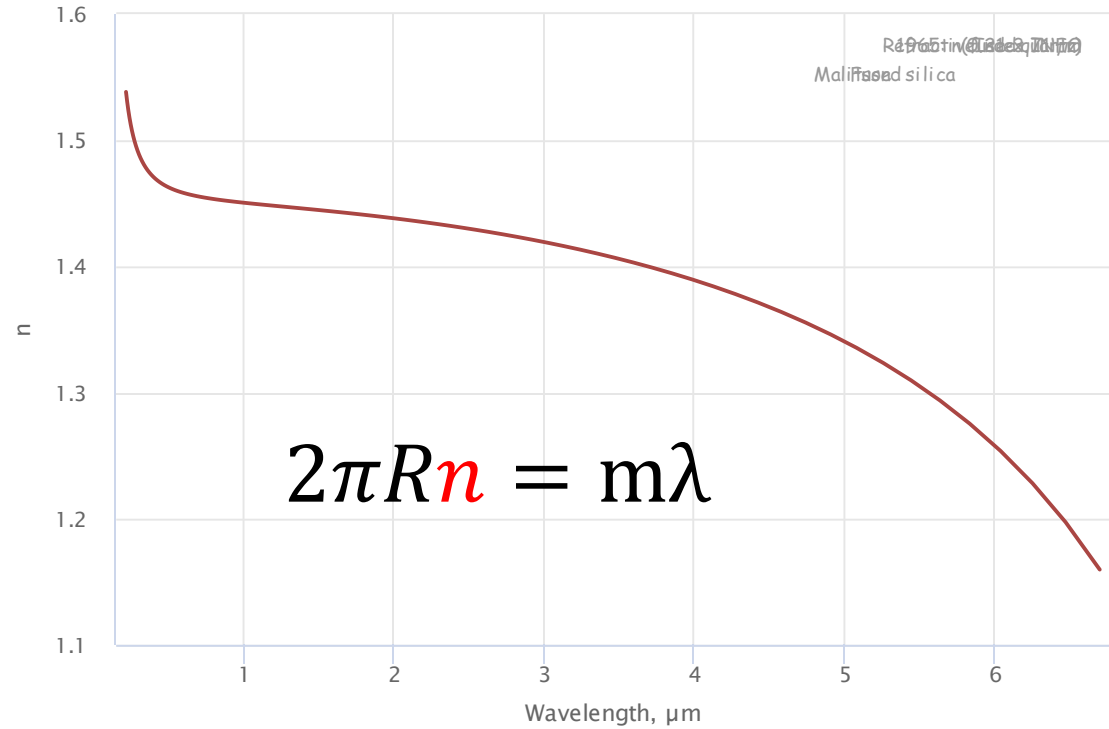
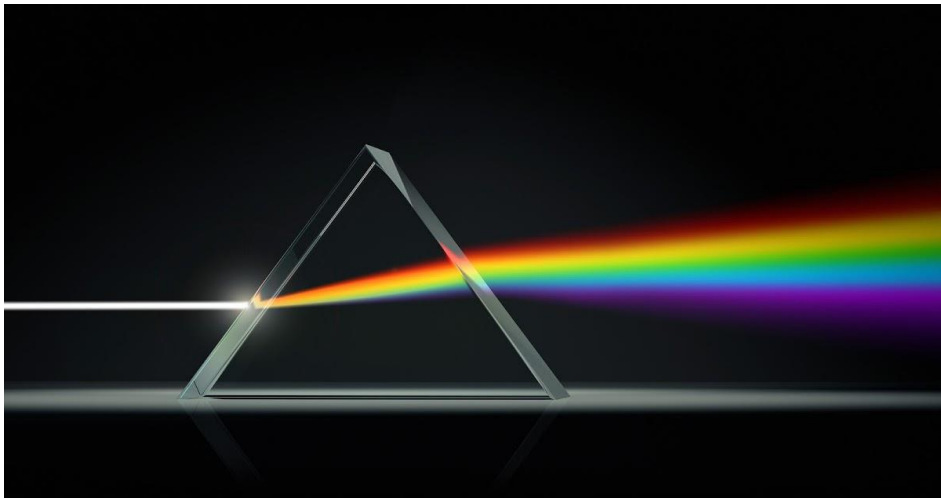


Material dispersion, Sellmeier's equation (for SiO₂)



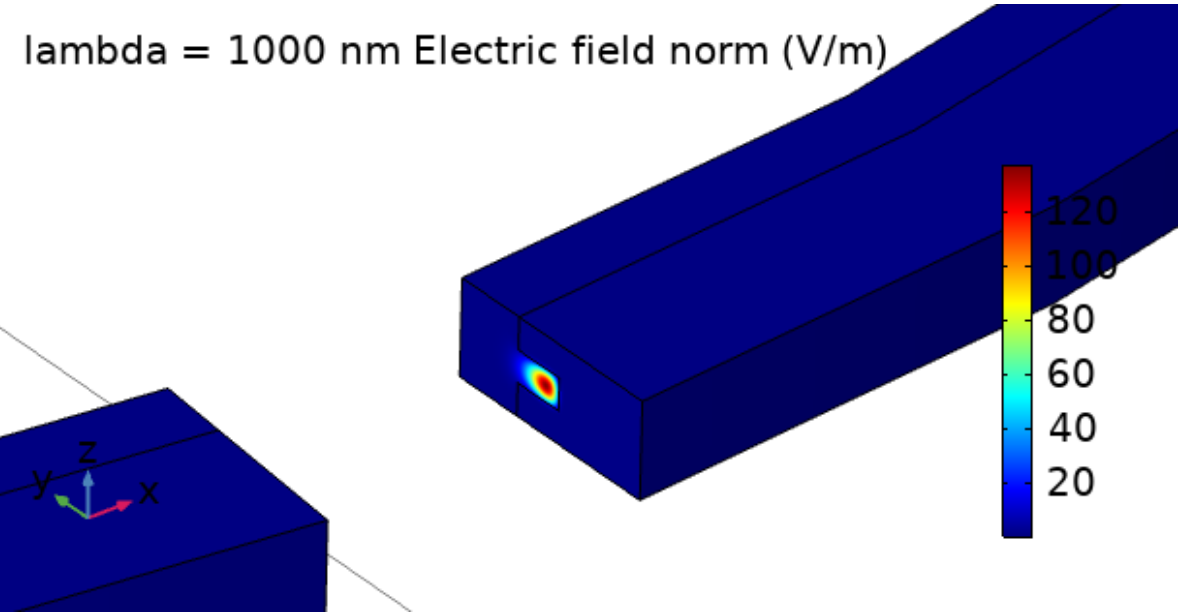
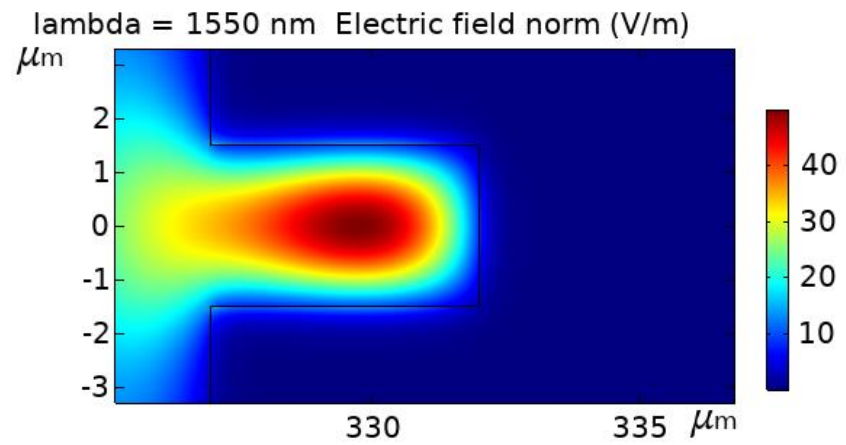
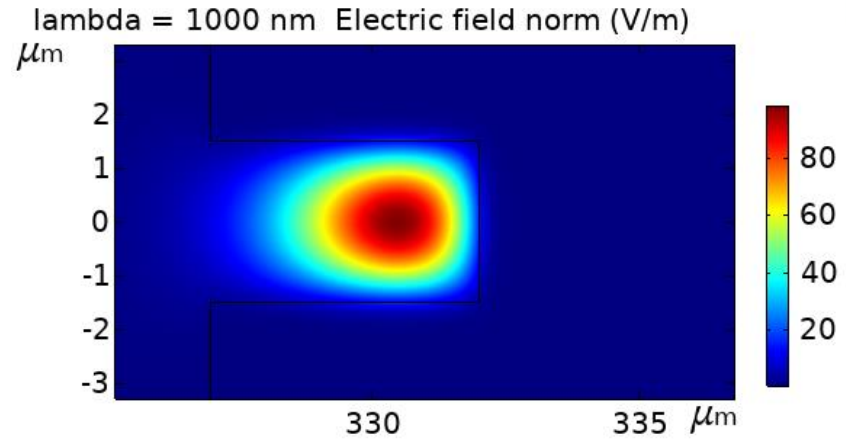
$$n^2 - 1 = \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161^2}$$

Material dispersion, Sellmeier's equation (for SiO₂)



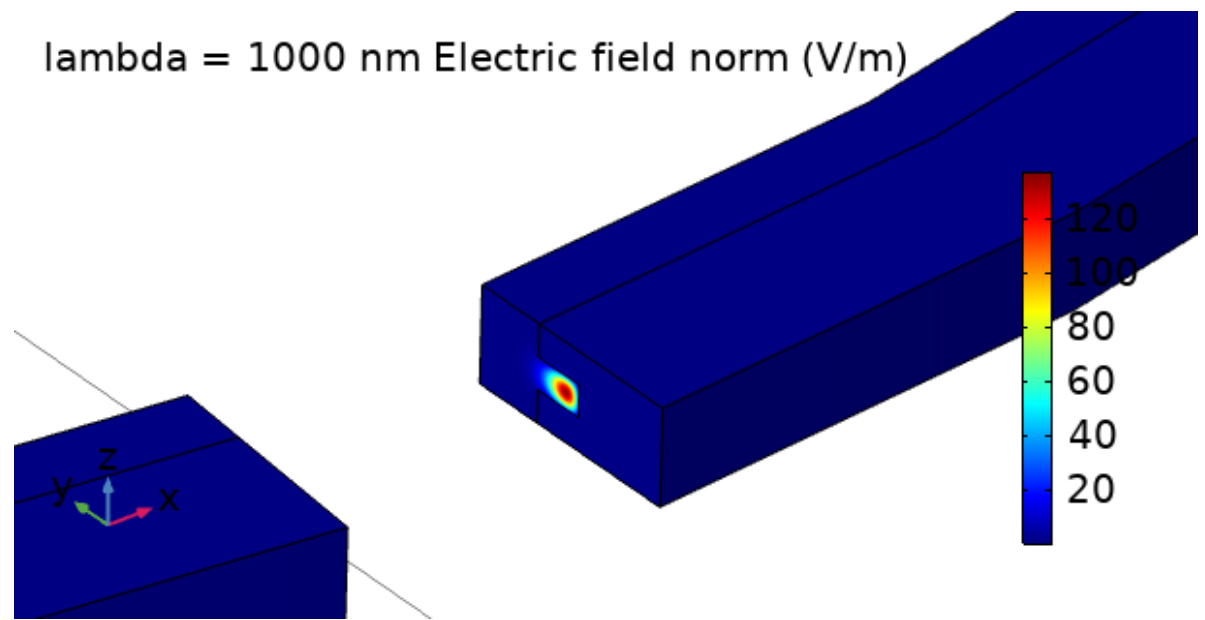
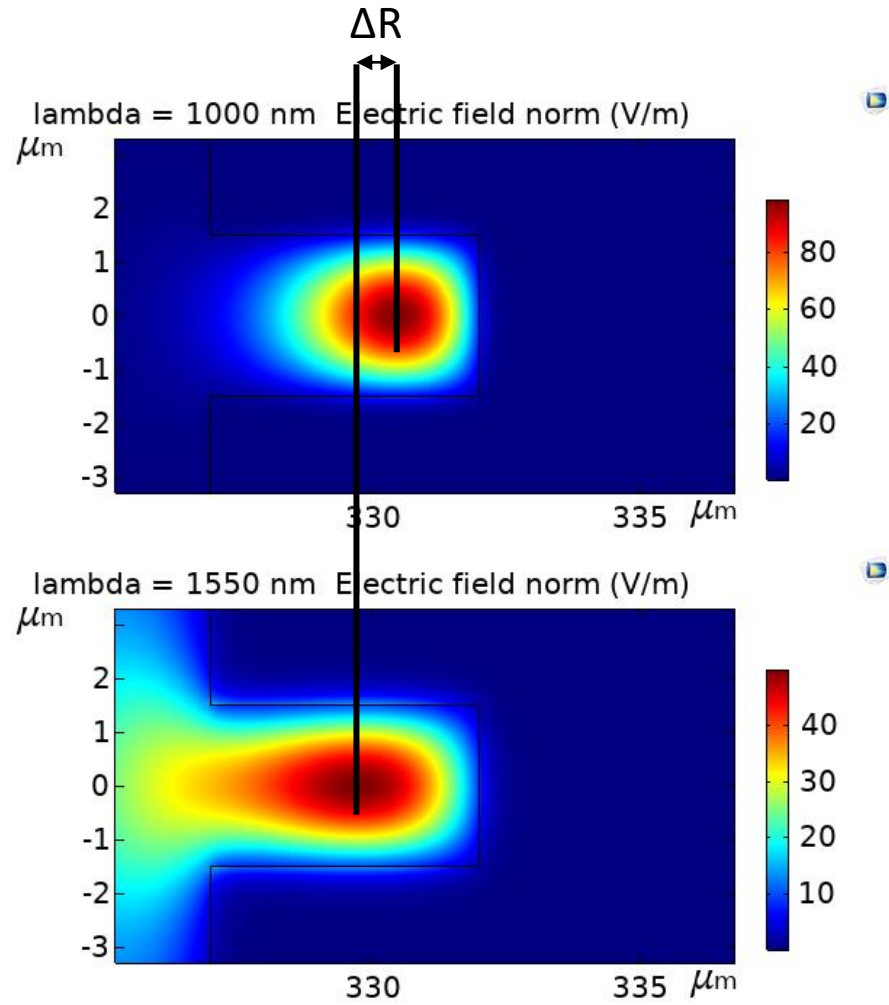
$$n^2 - 1 = \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161^2}$$

Geometric dispersion



$$2\pi R n = m\lambda$$

Geometric dispersion



$$2\pi R n = m\lambda$$

Dispersion analysis using Finite Element Method (FEM) simulations

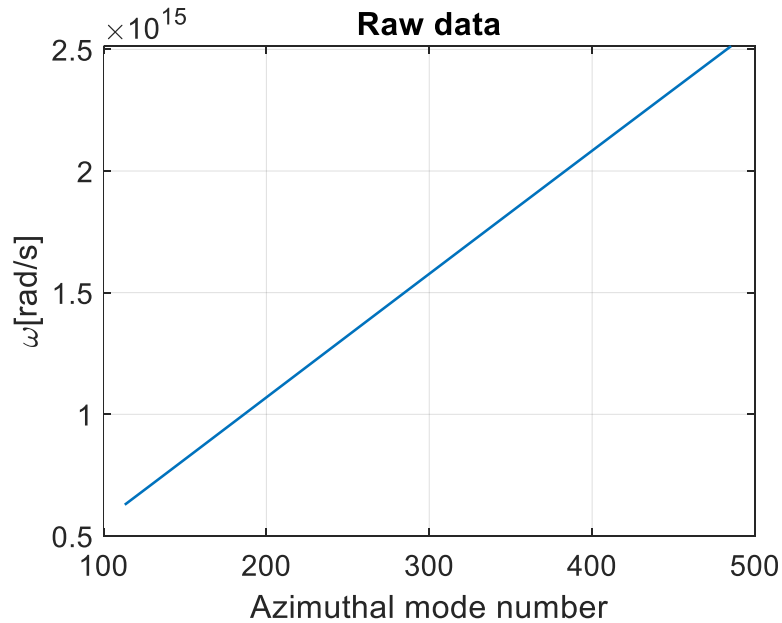
» Expression	Unit	Description
ewfd.freq	Hz	Frequency
ewfd.neff/r1	1/m	Refractive index n
Esq^2/Equa	um^2	Mode area
Esqr/Esq	μm	Effective radius
ewfd.neff*2*pi*ewfd.freq/c_const	1/m	m

freq	n	Aeff	Reff	m
1E+14	1.3187878	31.39729	317.362	917.639695
1.04E+14	1.3238103	29.66119	317.7076	958.731791
1.08E+14	1.3283721	28.09027	318.0327	999.762435
1.12E+14	1.3325382	26.66243	318.3391	1040.74308
1.16E+14	1.3363614	25.35935	318.6285	1081.68292
1.2E+14	1.3398857	24.1657	318.9025	1122.58941
1.24E+14	1.3431473	23.06855	319.1623	1163.46863
1.29E+14	1.3461771	22.05695	319.409	1204.32561
1.33E+14	1.3490008	21.12151	319.6438	1245.16456
1.37E+14	1.3516407	20.25418	319.8674	1285.98895
1.41E+14	1.3541157	19.44801	320.0809	1326.80177
1.45E+14	1.3564423	18.69692	320.2848	1367.60551
1.49E+14	1.3586347	17.99563	320.4799	1408.40232
1.53E+14	1.3607053	17.3395	320.6667	1449.19403
1.57E+14	1.3626652	16.72445	320.8459	1489.98223
1.61E+14	1.3645240	16.14687	321.0179	1530.76829
1.65E+14	1.3662902	15.60355	321.1832	1571.55341
1.69E+14	1.3679713	15.09162	321.3421	1612.33864
1.73E+14	1.3695742	14.60855	321.4952	1653.12488
1.78E+14	1.3711048	14.15204	321.6427	1693.91296

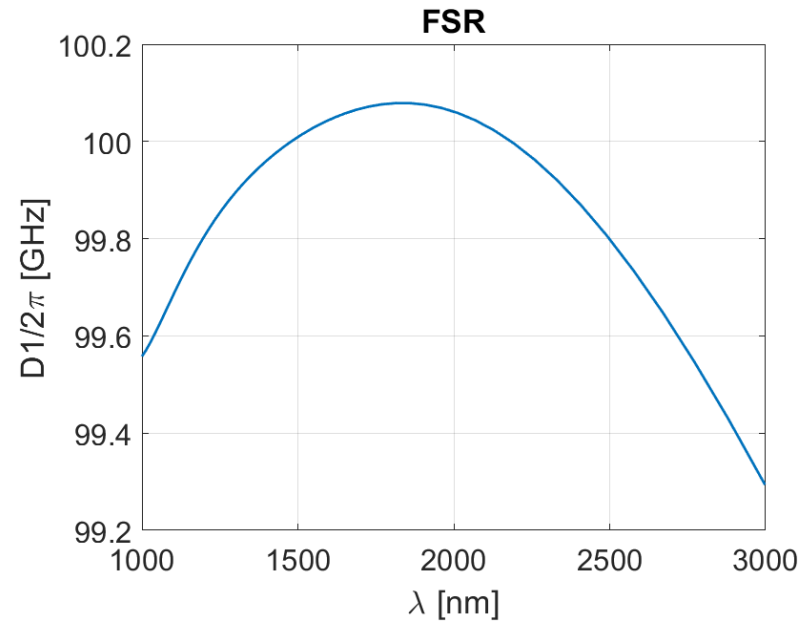
$$FSR = D_1 = \left. \frac{\partial \omega}{\partial m} \right|_{m_0} \text{ [rad/s]}$$

$$D_2 = \left. \frac{\partial^2 \omega}{\partial m^2} \right|_{m_0} \text{ [rad/s]}$$

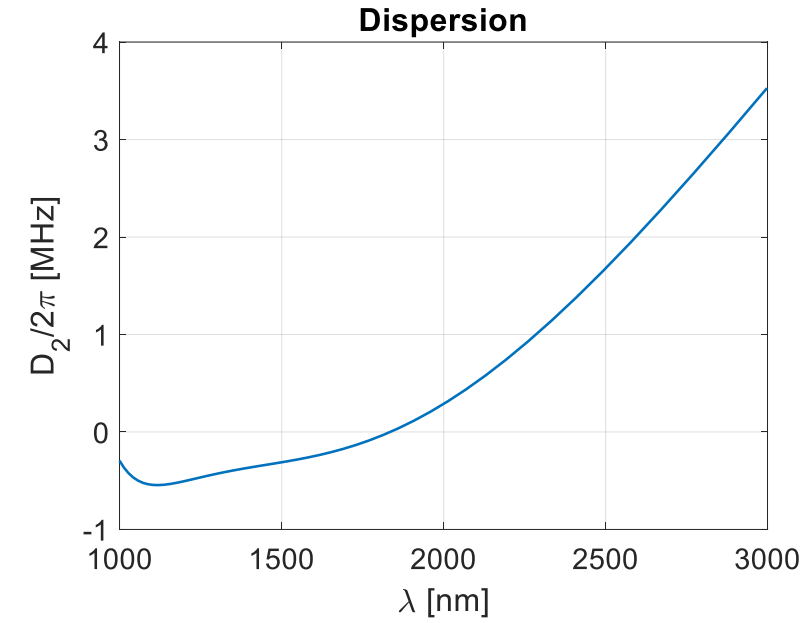
Dispersion analysis using Finite Element Method (FEM) simulations



$$\omega = \omega(m) \text{ [rad/s]}$$

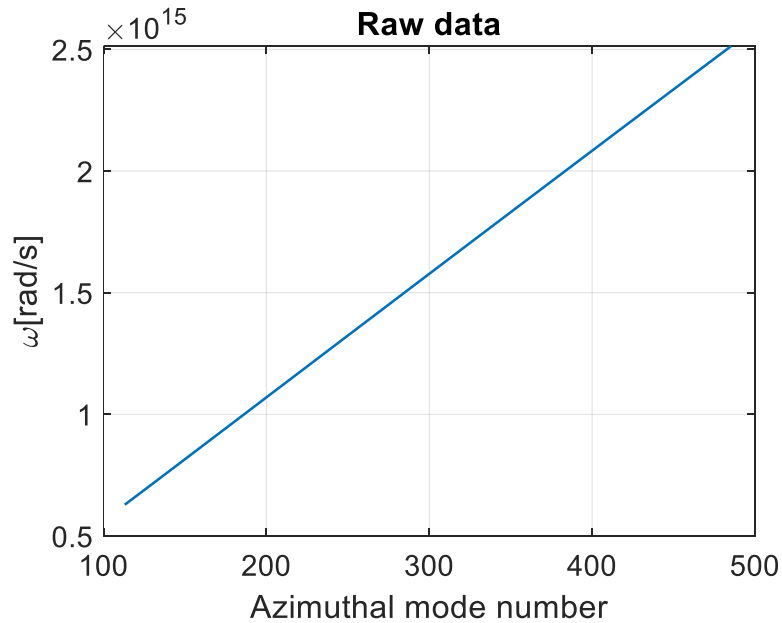


$$FSR = D_1 = \left. \frac{\partial \omega}{\partial m} \right|_{m_0} \text{ [rad/s]}$$

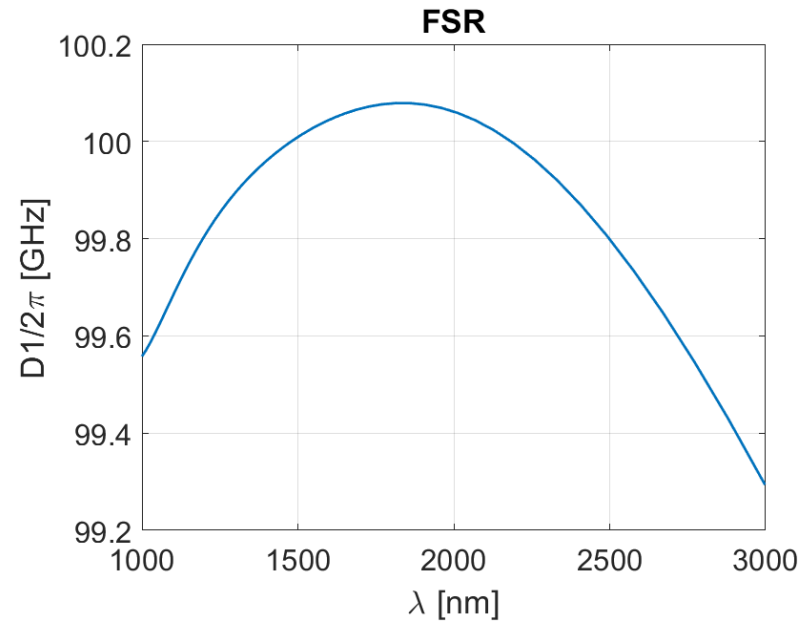


$$D_2 = \left. \frac{\partial^2 \omega}{\partial m^2} \right|_{m_0} \text{ [rad/s]}$$

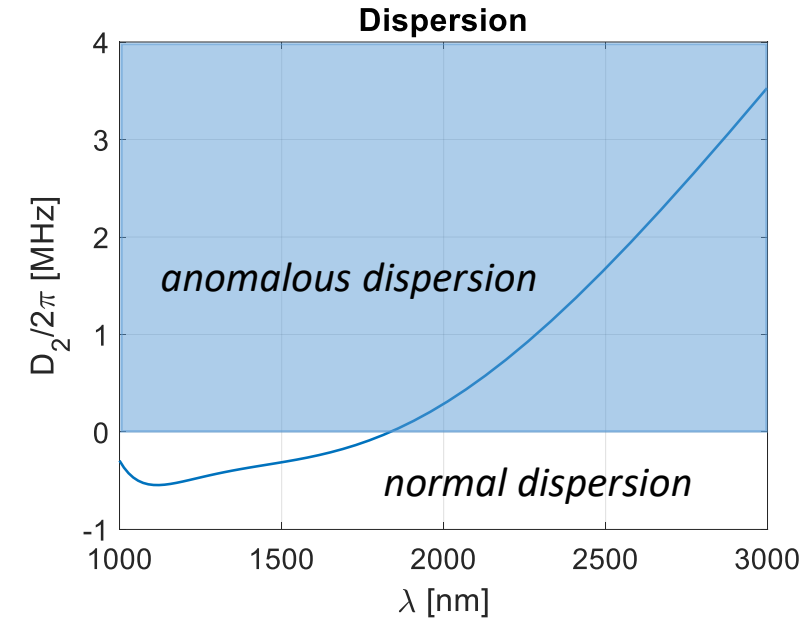
Dispersion analysis using Finite Element Method (FEM) simulations



$$\omega = \omega(m) \text{ [rad/s]}$$

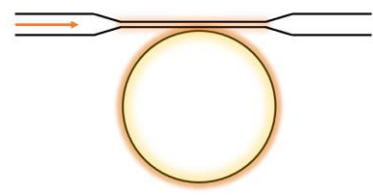


$$FSR = D_1 = \left. \frac{\partial \omega}{\partial m} \right|_{m_0} \text{ [rad/s]}$$

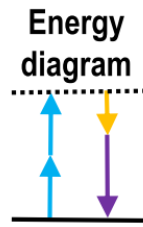
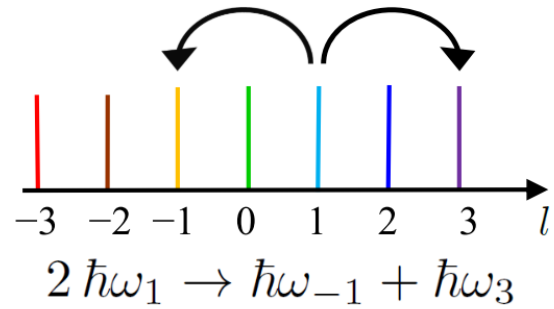


$$D_2 = \left. \frac{\partial^2 \omega}{\partial m^2} \right|_{m_0} \text{ [rad/s]}$$

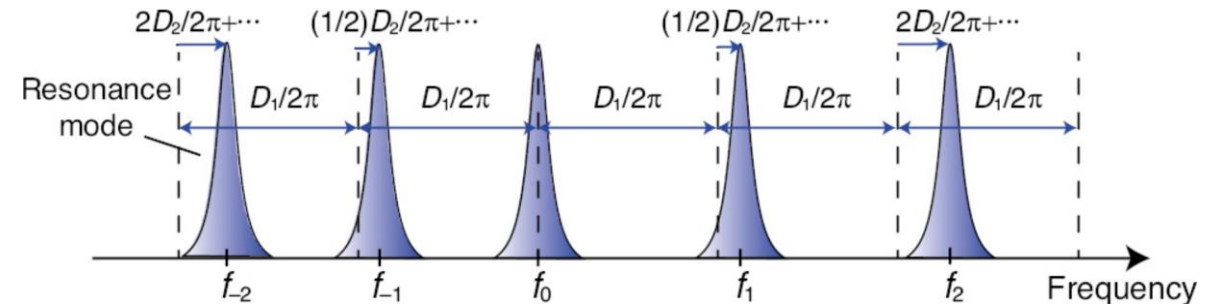
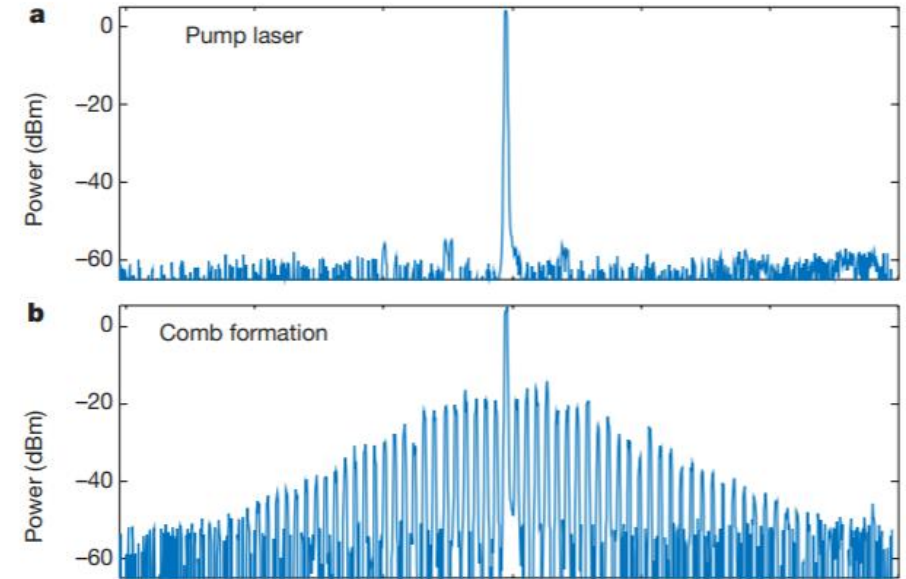
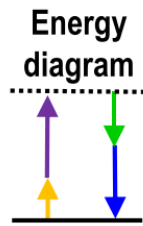
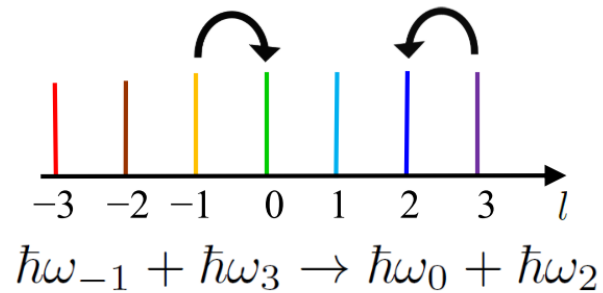
Four Wave Mixing (FWM), anomalous dispersion [4, 5, 6]



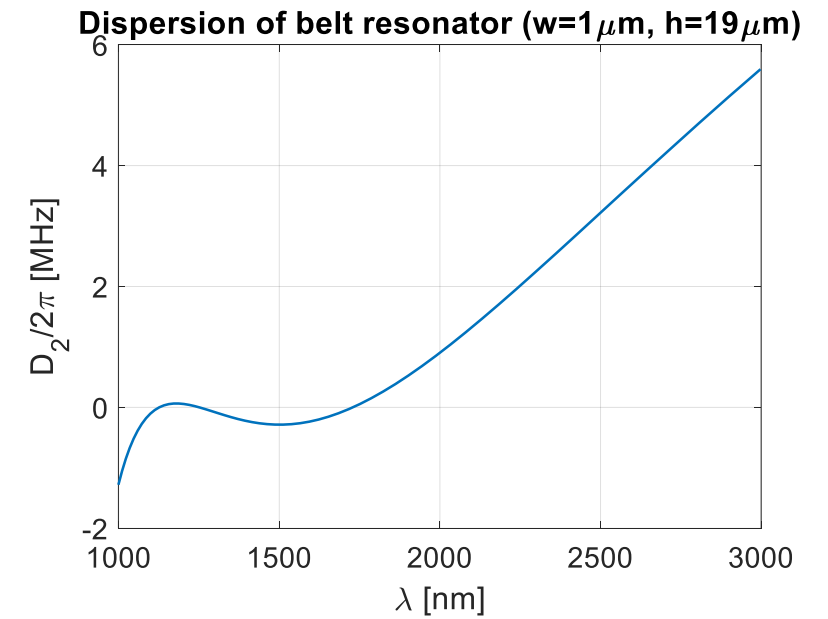
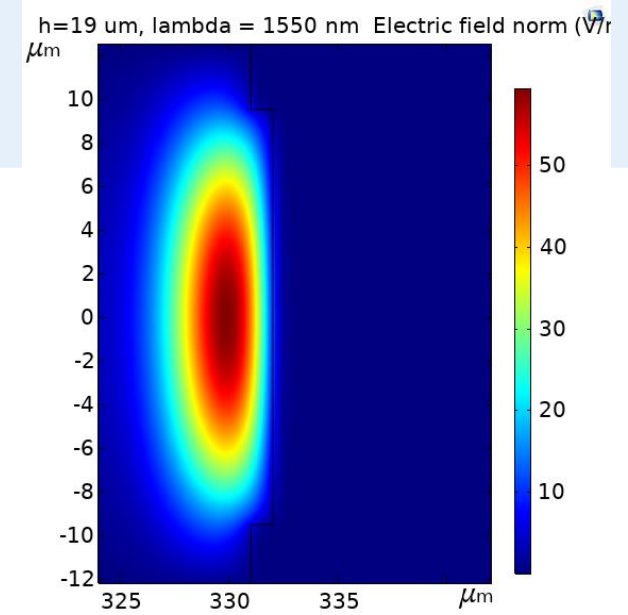
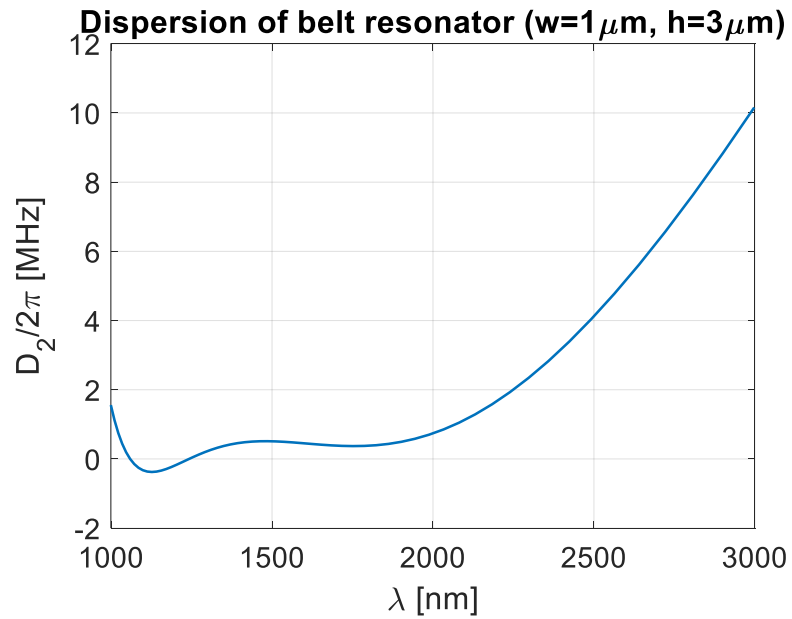
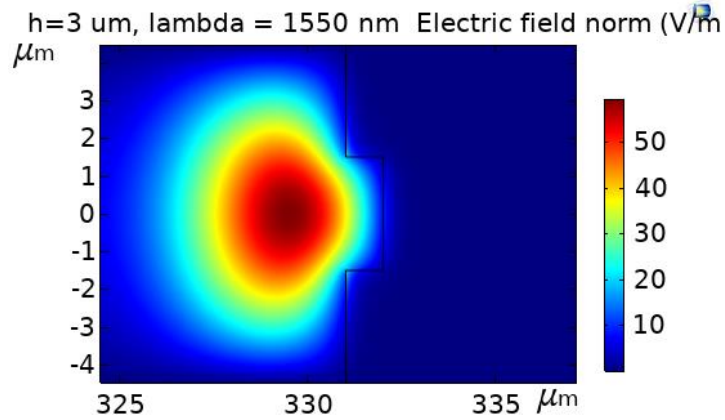
(a) Degenerate FWM



(b) Non degenerate FWM

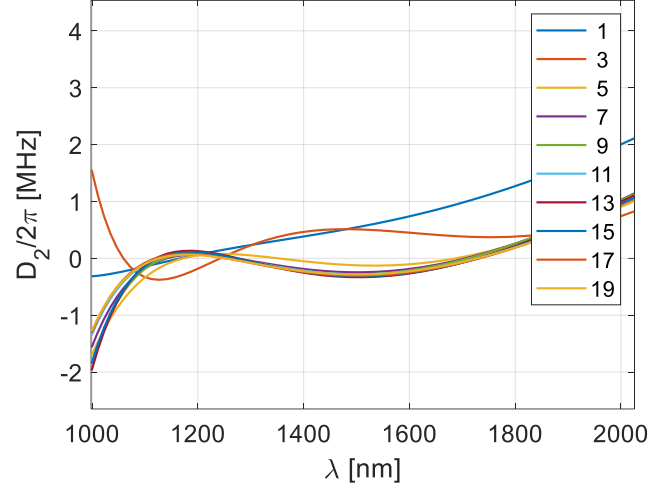


Dispersion engineering

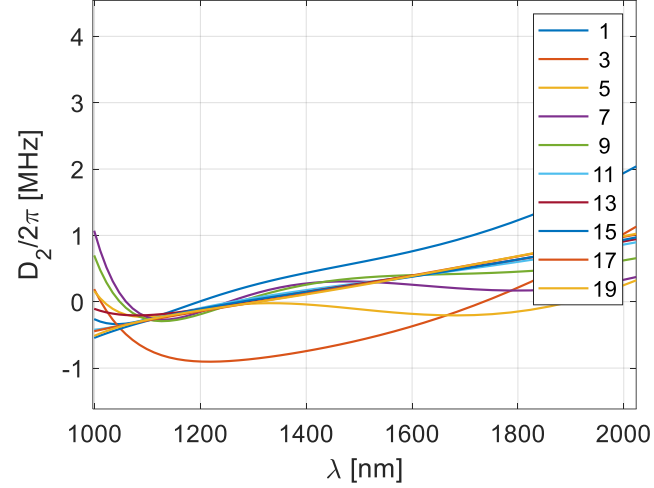


Dispersion engineering

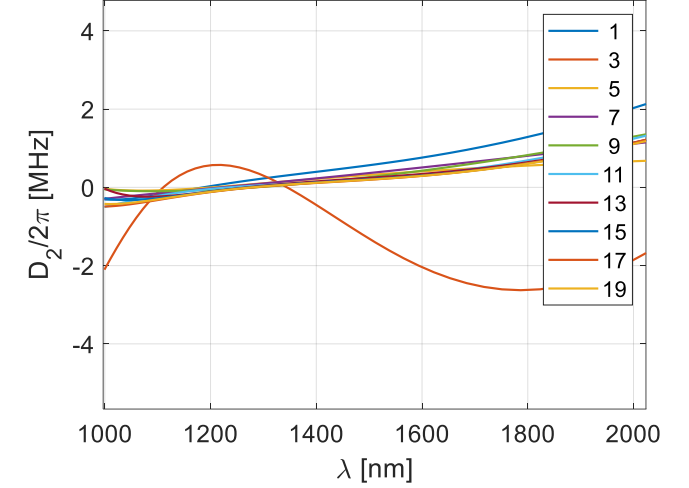
Dispersion of belt resonator ($w=1\mu\text{m}$, $h=[]\mu\text{m}$)



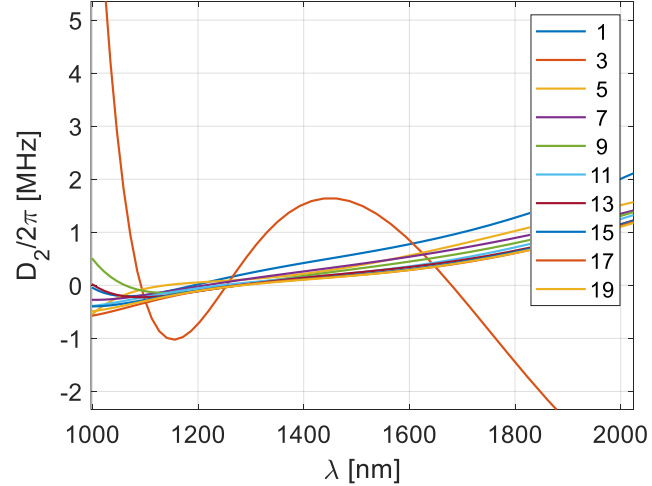
Dispersion of belt resonator ($w=3\mu\text{m}$, $h=[]\mu\text{m}$)



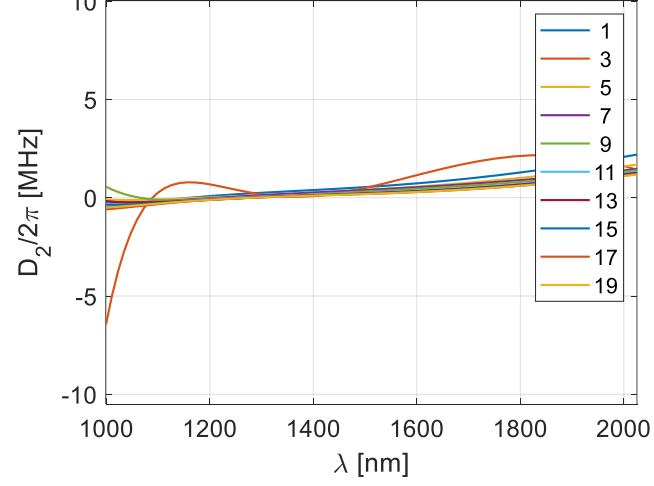
Dispersion of belt resonator ($w=5\mu\text{m}$, $h=[]\mu\text{m}$)



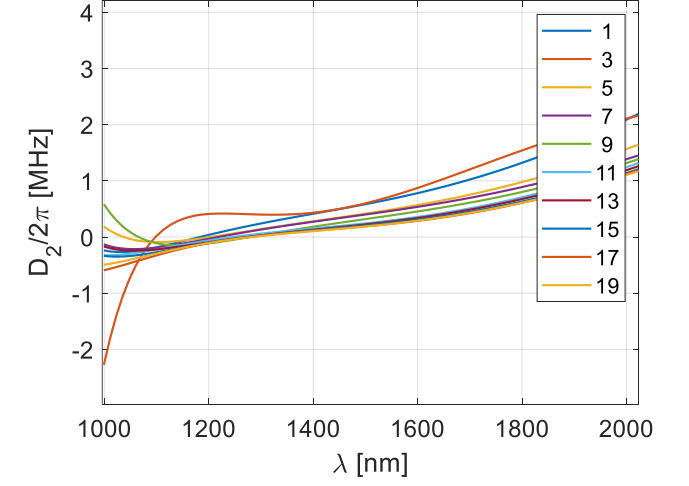
Dispersion of belt resonator ($w=7\mu\text{m}$, $h=[]\mu\text{m}$)



Dispersion of belt resonator ($w=9\mu\text{m}$, $h=[]\mu\text{m}$)



Dispersion of belt resonator ($w=11\mu\text{m}$, $h=[]\mu\text{m}$)



Modeling combs in pyLLE [7]

$$t_R \frac{\partial E(t, \tau)}{\partial t} = - \left(\frac{\alpha'}{2} - i\delta_0 \right) E + i \cdot \text{FT}^{-1} \left[-t_R D_{int}(\omega) \cdot \text{FT} [E(t, \tau)] \right] + \gamma |E|^2 E + \sqrt{\theta} E_{in}$$

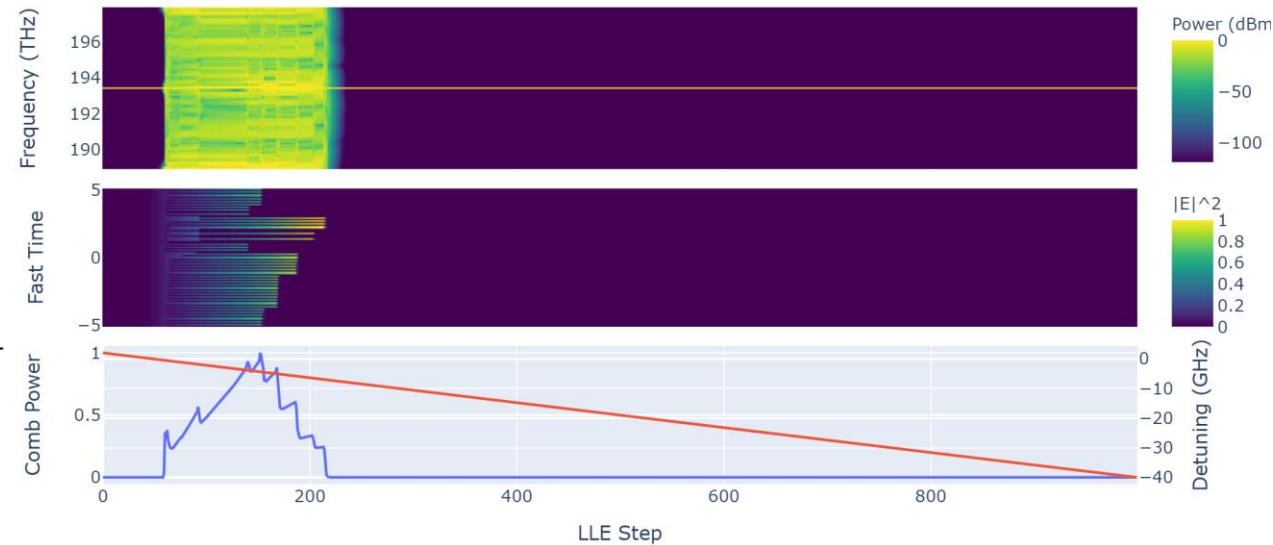
-- Solving standard LLE --

Simulation Parameters

R = 332.00 μm
 Qi = 10.00 M
 Qc = 10.00 M
 $\gamma = 1.55$

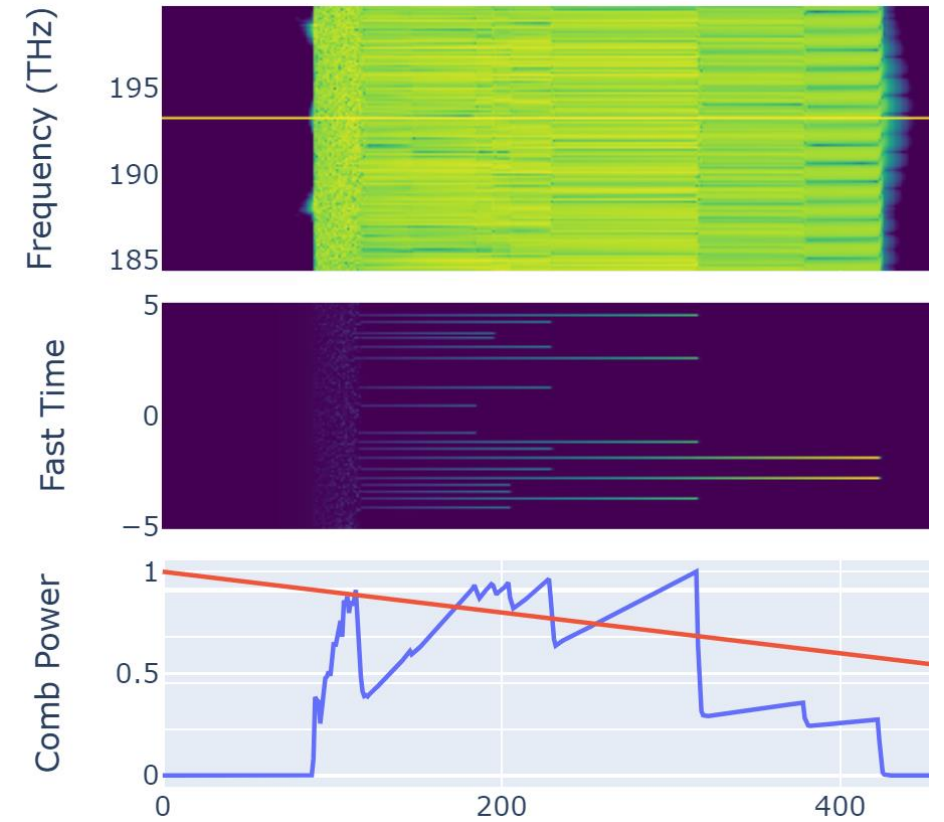
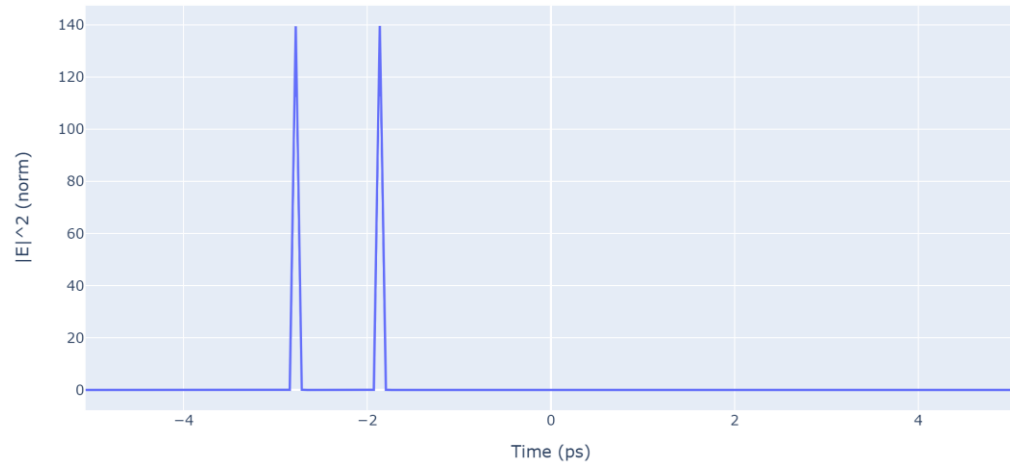
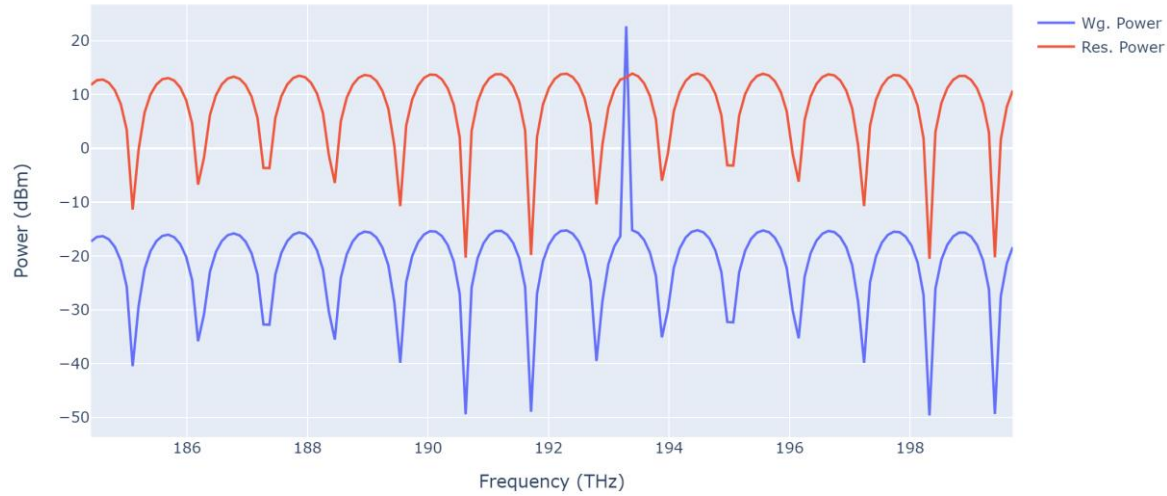
Simulation Parameters

Pin = 190.00 mW
 Tscan = 0.50 x1e6 Round Tri
 f_pmp = 193.41 THz
 $\delta\omega_{init} = 2.00 \times 2\pi$ GHz
 $\delta\omega_{end} = -40.00 \times 2\pi$ GHz
 $\mu_{sim} = [-45.00, 45.00]$
 $\mu_{fit} = [-45.00, 45.00]$



1	1850,188846679971364.00
2	1851,188945790329305.00
3	1852,189044900821708.00
4	1853,189144011447497.00
5	1854,189243122205595.00
6	1855,189342233094932.00
7	1856,189441344114439.00
8	1857,189540455263051.00
9	1858,189639566539706.00
10	1859,189738677943345.00
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12	1861,189936901127361.00
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28	1877,191522703735406.00
29	1878,191621817351810.00

Modeling combs in pyLLE



References

- [1] https://www.anglicannews.org/ImageGen.ashx?image=/media/1863503/garry-knight-flickr_st-pauls-cathedral-london-england_700x467.jpg
- [2] <https://www.standard.co.uk/s3fs-public/thumbnails/image/2017/10/11/15/stpaulsinterior1110b.jpg>
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- [4] Chembo, Y. K. (2016). Kerr optical frequency combs: Theory, applications and perspectives. *Nanophotonics*, 5(2), 214–230.
- [5] Stern, B., Ji, X., Okawachi, Y., Gaeta, A. L., & Lipson, M. (2018). Battery-operated integrated frequency comb generator. *Nature*, 562(7727), 401–405. <https://doi.org/10.1038/s41586-018-0598-9>
- [6] S. Fujii, T. Tanabe, (2020) Dispersion engineering and measurement of whispering gallery mode microresonator for Kerr frequency comb generation, *Nanophotonics* 9, 1087–1104
- [7] Moille, G., Li, Q., Lu, X., & Srinivasan, K. (2019). Pylle: A fast and user friendly lugiato-lefever equation solver. *Journal of Research of the National Institute of Standards and Technology*, 124(124012).



Thank you for attention!

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