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May 17 – 20, 2016, Rīga, Latvia

Abstracts

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Welcome Address

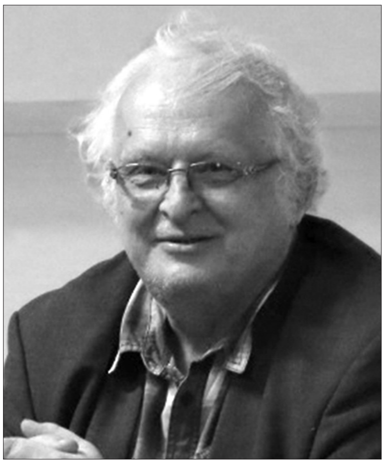
Dear Colleagues,

On behalf of the Organizing Committee, we welcome you to the 10th International Conference of the Progress on Difference Equations taking place in the Riga, Latvia!

The aim of the Conference is to bring together members of the mathematical community. This conference, held under the auspices of the International Society for Difference Equations, aims to be a forum where researchers can share their work and discuss the latest developments in the areas of difference equations, discrete dynamical systems and their applications. About 40 researchers, university academic staff members, and students of Mathematics have applied for the participation. They represent leading institutions of research and Higher education of Belarus, Brazil, Czech Republic, France, Germany, India, Italy, Poland, Portugal, Romania, Spain, Serbia, UAE, USA, and, of course, of Latvia.

We wish you a scientifically stimulating and enjoyable time in Riga!

Inese Bula,
Chair of Organizing Committee



Andrejs Reinfelds,
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University of Latvia, Rīga, Latvia

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LIST OF SPEAKERS

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CONTRIBUTED TALKS

Plenary Talks

Abstracts of PODE 2016, May 17–20, 2016, Rīga, Latvia

GLOBAL DYNAMICS OF DIFFERENCE EQUATIONS: APPLICATIONS TO POPULATION DYNAMICS

SABER ELAYDI

Trinity University

San Antonio, Texas, USA

E-mail: selaydi@trinity.edu

In this talk we will present the latest development in the global dynamics of two types of systems generated by triangular maps and monotone maps. The dynamics of planar monotone maps have been well understood through the work of Hal Smith. The theory of monotone maps is now extended to higher dimensional maps via geometrical interpretation of monotonicity. Another class of maps for which the Global dynamics have been successfully established, is the class of triangular maps where the Jacobian matrix of the map is triangular. Applications of our theory to population biology will be presented. For instance, hierarchical models may be represented by triangular maps defined on R_+^k. In particular, we focus our attention on models with the Allee effect. The general theory of the global dynamics of triangular maps was established by Balreira, E., and Luis [1]. Here we extend these results to include the difficult case of non-hyperbolic maps, building upon the work by Assas et al. [2,3]. We show that in the case of non-hyperbolic maps, the center manifold is semi-stable from above. Finally, we show how immigration to one of the species or to both would change drastically the dynamics of the system. It is shown that if the level of immigration to one or both species is above a specified level, then there will be no extinction region.

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ENTROPY LOCKING

MICHAŁ MISIUREWICZ

Indiana University-Purdue University Indianapolis

402 N. Blackford Street, Indianapolis, IN 46202, USA

E-mail: mmisiure@math.iupui.edu

Consider discontinuous piecewise linear interval maps with two pieces, where the map is increasing on one piece and decreasing on the other piece. Often the topological entropy depends only on the slopes, not on the size of the jump at the discontinuity point. We present a simple explanation of this phenomenon.

This is joint work with David Cosper.

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STABILITY OF SWITCHED DIFFERENCE EQUATIONS ON ORDERED SPACES

STEFAN SIEGMUND

Center for Dynamics & Institute for Analysis, Department of Mathematics

TU Dresden, 01062 Dresden, Germany

E-mail: `stefan.siegmund@tu-dresden.de`

Let $A_1, \dots, A_N \in \mathcal{L}(X)$ be $N \in \mathbb{N}$ bounded linear operators on a Banach space X. For an arbitrary *switching signal* $\sigma : \mathbb{N}_0 \to \{1, \dots, N\}$ consider the *linear switched difference equation*

$$x_{k+1} = A_{\sigma(k)} x_k \quad \text{for } k \in \mathbb{N}_0. \tag{1}$$

If all operators leave a cone $K \subseteq X$ invariant, i.e. $A_i K \subseteq K$, then (1) is called positive.

We provide sufficient criteria for the stability of positive linear switched systems (1). More precisely, we assume the existence of an interior point in the cone which behaves well under the action of every single operator A_i. Our main tool for the proof is an extension of the concept of linear Lyapunov functions for positive systems to the setting of infinite-dimensional partially ordered spaces. We illustrate our results with examples.

This is joint work with Doan Thai Son, Anke Kalauch and Markus Klose [1].

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ON SOME SOLVABLE CLASSES OF DIFFERENCE EQUATIONS AND SYSTEMS OF EQUATIONS

STEVO STEVIĆ

Mathematical Institute of the Serbian Academy of Sciences

Knez Mihailova 36/III, 11000 Beograd, Serbia

E-mail: `sstevic@ptt.rs`

BRATISLAV IRIČANIN

Faculty of Electrical Engineering, Belgrade University

Bulevar Kralja Aleksandra 73, 11000 Beograd, Serbia

E-mail: `iricanin@etf.rs`

The old area of solving difference equations and systems has re-attracted some recent attention. Our idea of transforming complicated difference equations into simpler solvable ones, used in [5] for explaining the solvability of the equation appearing in [3], was later essentially employed and developed in numerous other papers (e.g. in [1], [2], [4], [6]-[11]). Another area of some recent interest, essentially initiated by G. Papaschinopoulos and C. J. Schinas, is studying symmetric and close to symmetric systems of difference equations. Among these types of systems there are also some solvable ones (e.g. the ones in [2], [6]-[11]). In this talk we will present some new classes of difference equations and systems of difference equations solvable in closed form and briefly describe some methods for getting formulas for their solutions. Also we will explain what essentially stands behind the solvability of the equations and systems. Beside real difference equations and systems we will also discuss some equations and systems with complex coefficients and initial values.

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Contributed Talks

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GLOBAL STABILITY IN A DISCRETE-TIME CONTEST-COMPETITION MODEL

ZIYAD ALSHARAWI

American University of Sharjah

Sharjah, UAE

E-mail: zsharawi@aus.edu

In this talk, we consider a general discrete-time model and investigate its dynamics under the effect of constant effort exploitation. We show global stability under certain conditions on the recruitment function and the harvesting parameter. Also, we discuss the conditions under which a Neimark-Sacker bifurcation occurs.

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PERIODICITY OF SOME RATIONAL DIFFERENCE EQUATIONS WITH A POSITIVE REAL POWER

AIJA ANISIMOVA

University of Latvia, Department of Mathematics

Zeļļu iela 8, Rīga LV-1002, Latvia

E-mail: aija.anisimova@gmail.com

In this talk we consider the existence of periodic solutions of some second order rational difference equations in the form (1):

$$x_{n+1} = \frac{\alpha + \beta x_n^k + \gamma x_{n-1}^k}{A + Bx_n^k + Cx_{n-1}^k}, \quad n = 0, 1, 2, .., \tag{1}$$

with nonegative parameters $\alpha, \beta, \gamma, A, B, C$ and arbitrary nonegative initial conditions x_{-1}, x_0 such that the denominator is always positive and the two arguments x_n and x_{n-1} are raised to a positive real power $k \in (0, \infty)$.

The boundedness character of solutions of the Eq.(1) have been studied in [1]; furthermore, a number of interesting open problems are also posed in [1].

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CONTINUOUS DEPENDENCE ON DATA FOR THE SOLUTIONS OF SOME DIFFERENTIAL AND DIFFERENCE EQUATIONS

NARCISA APREUTESEI

Department of Mathematics and Informatics Technical University of Iasi

Romania

E-mail: napreut@gmail.com

We are interested in some classes of differential and difference equations associated with maximal monotone operators in Hilbert spaces. We present some continuous dependence results for their solutions on the operator that governs the equation.

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SOLUTIONS WITH PERIOD TWO

MARUTA AVOTINA

University of Latvia

Zeļļu iela 25, Rīga LV-1002, Latvia

E-mail: maruta.avotina@lu.lv

We investigate the behaviour of solutions for homogeneous and non-homogeneous linear difference equations

$$x_{n+1} = A_1 x_n + A_2 x_{n-1} + \ldots + A_{k+1} x_{n-k} + B \tag{1}$$

that have a root -1 of the characteristic equation.

The linearized equation associated with (1) about the equilibrium point $\bar{x}$ and the characteristic equation in both cases (homogeneous and non-homogeneous) are the same and in the following form

$$y_{n+1} = A_1 y_n + A_2 y_{n-1} + \ldots + A_{k+1} y_{n-k}, \tag{2}$$

$$\lambda^{k+1} - A_1 \lambda^k - A_2 \lambda^{k-1} - \ldots - A_{k+1} = 0, \tag{3}$$

although the homogeneous equation always has an equilibrium $\bar{x} = 0$, but the non-homogeneous equation has a non-zero equilibrium $\bar{x} = \frac{B}{1-A_1-A_2-\ldots-A_{k+1}}$.

If the solution converges to the period two solution then in some cases it is possible to express this period two solution in terms of initial conditions.

In many cases (not only for linear difference equations but also for some rational difference equations, see [1; 2; 3; 4; 5] the root -1 is connected with the period-two solution. The aim of the investigation is to determine why and how the root -1 of the characteristic equation affects the behaviour of solutions.

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INVARIANTS FOR A CLASS OF DISCRETE DYNAMICAL SYSTEMS GIVEN BY RATIONAL MAPPINGS

IGNACIO BAJO

Universidad de Vigo

Depto. Matemática Aplicada II, E.I. Telecomunicación, Campus Marcosende, 36310 Vigo (Spain)

E-mail: ibajo@dma.uvigo.es

An invariant or first integral of a discrete dynamical system $x(k+1) = F(x(k))$ with domain $\mathcal{D}$ is a non constant map $H : \mathcal{U} \subset \mathbb{K}^n \to \mathbb{K}$ defined in an open and dense subset $\mathcal{U}$ of $\mathcal{D}$ such that for all $x \in \mathcal{U}$ it holds $H(F(x)) = H(x)$.

In this work, we study the existence of invariants for a family of rational dynamical systems. Explicitly, let $\mathbb{K}$ denote either $\mathbb{R}$ or $\mathbb{C}$. We consider the discrete dynamical systems in an open domain $\mathcal{D}$ of $\mathbb{K}^n$ of the form

$$x(k+1) = F(x(k)) = (F_1(x(k)), \dots, F_n(x(k))), \quad x(k) \in \mathcal{D} \subset \mathbb{K}^n \tag{1}$$

where the functions $F_i : \mathcal{D} \subset \mathbb{K}^n \to \mathbb{K}$ are linear fractionals sharing denominator:

$$F_i(x) = \frac{a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + c_i}{b_1x_1 + b_2x_2 + \cdots + b_nx_n + d}, \;\; i = 1, 2, \dots, n,$$

for $x = (x_1, x_2, \cdots, x_n)$ and all involved parameters in $\mathbb{K}$. Such systems can be written with the aid of homogeneous coordinates as the composition of a linear map in $\mathbb{K}^{n+1}$ with a certain projection and their behaviour is strongly determined by the spectral properties of the corresponding linear map.

We will prove that if $n \geq 2$ then every system of this kind admits an invariant, both in the real and in the complex case. More precisely, our main result will be

Theorem 1. *Consider $n > 1$. If the dynamical system given by (1) is defined in a nonempty open set $\mathcal{D}$, then it admits an invariant defined in an open and dense subset*

$$\mathcal{U} = \{x \in \mathcal{D} \; : \; \mathcal{Q}(x) \neq 0\},$$

where $\mathcal{Q}(x)$ is a polynomial of degree 2 defined by a couple of (not necessarily distinct) eigenvectors u_1, u_2 of a matrix defined by the coefficients of the components of F.

In fact, for a sufficiently large n several functionally independent invariants can be obtained and, in many cases, the invariant can be chosen as the quotient of two quadratic polynomials. In this cases one has, as a consequence, that every orbit of the system results to be contained in a certain F-invariant quadric.

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ON DIFFERENCE EQUATIONS WITH PREDETERMINED FORBIDDEN SETS

FRANCISCO BALIBREA

Departamento de Matemáticas

Campus de Espinardo de la Universidad de Murcia, 30100, Murcia (Spain)

E-mail: balibrea@um.es

Let $f : \mathbb{R}^k \to \mathbb{R}$ be of the form $f(x_n, ..., x_{n-k+1}) = \frac{P(x_n, x_{n-1}, ..., x_{n-k+1})}{Q(x_n, x_{n-1}, ..., x_{n-k+1})}$ that is, a *rational function*. The corresponding *rational difference equation* of order k is given by

$$x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-k+1})$$

which can be seen as the following discrete dynamical system associate to the iteration function.

$$F(x_n, x_{n-1}, ..., x_{n-k+1}) = (f(x_n, x_{n-1}, ..., x_{n-k+1}), x_n, ..., x_{n-k+2})$$

A *solution* of the equation is the sequence of numbers $(x_n)_{n=0}^{\infty}$ where $(x_0, ..., x_{k-1}) \in \mathbb{R}^k$ is the given vector of *initial conditions*.

Some vectors of initial conditions do not allow to construct a solution because there is a member x_{n+1} of the solution that can not be defined, usually because $Q(x_n, ..., x_{n-k+1}) = 0$. It can also happen by the effect of negative parameters in the equation. We will call *forbidden set* of the equation to the set of vectors $X \in \mathbb{R}^k$ for which the solution taking them as initial conditions are not defined. It will be denoted by $\mathfrak{F}$. Given a rational difference equation, it is a classical problem to construct its forbidden set.

In this talk we will deal with the converse problems. Given a set $\mathfrak{F}$, find an iteration function in such a way that its associate $\mathfrak{F}$ has been previously specified. For example, given an arbitrary closed set $C \subset \mathbb{R}$, we are able to construct even a non-autonomous rational difference equation with a forbidden set holding $\overline{\mathfrak{F}} = C$.

On other hand, in some families of difference equations, for all its members $\mathfrak{F}$ always contains non-bounded hypersurfaces and it is impossible to use the constructions used in the cases considered in the former paragraph.

We will also deal with some generalizations of the same problems on the forbidden sets in the setting of systems of difference equations, equations with complex parameters and equations outside of the rational frame. Some type of universal behaviour will be also presented.

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POSITIVE AND OSCILLATING SOLUTIONS OF DISCRETE LINEAR EQUATIONS WITH A SINGLE DELAY

JAROMÍR BAŠTINEC and JOSEF DIBLÍK

Brno University of Technology

Technická 10, 616 00 Brno, Czech Republic

E-mail: `bastinec@feec.vutbr.cz, diblik@feec.vutbr.cz`

In the talk we deal with positive and oscillating solutions of linear difference equations

$$\Delta x(n) = -p(n)x(n-k)$$

where $p\colon Z_a^\infty \to \mathbb{R}^+ := (0,\infty)$, $\mathbb{Z}_s^q := \{s, s+1, \dots, q\}$, $k \geq 1$, a is an integer and $n \in \mathbb{Z}_a^\infty$.

We give sufficient conditions with respect to the right-hand side of equation to guarantee the existence of at least one initial function

$$x(n) = \varphi^*(n),\ n \in \mathbb{Z}_{a-k}^a$$

with $\varphi^*\colon \mathbb{Z}_{a-k}^a \to (0,\infty)$ such that the solution $x^* = x^*(n; a, \varphi^*)$ remains positive on $\mathbb{Z}_{a-k}^\infty$.

This problem is solved utilizing results on difference inequalities (e.g., [1, Theorem 3] and [2, Theorem 7.6.2]). Main assumption on p is $0 < p(n) \leq p_\ell(n)$ for a fixed $\ell \geq 0$ and for all sufficiently large $n \to +\infty$ where

$$p_\ell(n) := \left(\frac{k}{k+1}\right)^k \left[\frac{1}{k+1} + \frac{k}{8n^2} + \frac{k}{8(n\ln n)^2} + \dots + \frac{k}{8(n\ln n\dots\ln_\ell n)^2}\right].$$

We show that, in the case of an opposite inequality for $p(n)$, all solutions of the equation considered are oscillating for $n \to \infty$. This is proved with the aid of [3, Theorem 4, p. 66].

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ASYMPTOTICS OF SOLUTIONS OF PERTURBATIONS OF DIFFERENCE EQUATIONS WITH A NONUNIFORM EXPONENTIAL DICHOTOMY

SIGRUN BODINE

University of Puget Sound

Tacoma, WA, 98416, USA

E-mail: `sbodine@pugetsound.edu`

For invertible $d \times d$ matrices $A(n)$, we are interested in the asymptotic behaviour of solutions of difference equations

$$y(n+1) = A(n)y(n) + f(n, y(n)), \tag{1}$$

where $n \in \mathbb{N}$ or $n \in \mathbb{Z}$. Here we assume that the unperturbed systems $x(n+1) = A(n)x(n)$ satisfies a nonuniform exponential dichotomy (see, e.g., [1]), i.e., there exist constants $\alpha, K > 0$, $\varepsilon \geq 0$, and a projection P such

$$\begin{aligned} \left|X(n)PX^{-1}(k)\right| &\leq Ke^{-\alpha(n-k)+\varepsilon|k|}, \qquad k \leq n, \\ \left|X(n)QX^{-1}(k)\right| &\leq Ke^{-\alpha(k-n)+\varepsilon|k|}, \qquad k > n, \end{aligned}$$

where $Q = Id - P$ is the complementary projection. Depending on the size of the perturbation $f(n, y(n))$, we will discuss results on the asymptotics of solutions of (1).

Our work was motivated by a recent publication [2] concerning the continuous case
$y' = A(t)y + f(t, y)$.

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ON ATTRACTORS IN DYNAMICAL SYSTEMS ARISING IN GENE REGULATORY NETWORK THEORY

EDUARD BROKAN[1] and FELIX SADYRBAEV[2]

[1] *Daugavpils University*
Parades str. 1, Daugavpils, LV-5400, Latvia
[2] *Institute of Mathematics and Computer Science*
Raiņa bulvāris 29, Rīga LV-1459, Latvia
E-mail: `felix@latnet.lv`

We consider systems of differential equations occuring in the theory of telecommunications/gene regulatory systems. In the theory of such systems an important role play attractor selection and attracting sets ([1], [2]). The configuration of a network and interrelations between nodes heavily depend on the structure of attractors. We consider the dynamical system

$$\begin{cases} x_1' = \dfrac{1}{1+e^{-\mu(W_{11}x_1+\ldots+W_{1n}x_n-\theta)}} - x_1, \\ x_2' = \dfrac{1}{1+e^{-\mu(W_{21}x_1+\ldots+W_{2n}x_n-\theta)}} - x_2, \\ \cdots \quad \cdots \quad \cdots, \\ x_n' = \dfrac{1}{1+e^{-\mu(W_{n1}x_1+\ldots+W_{nn}x_n-\theta)}} - x_n, \end{cases}$$

where W_{ij} are entries of the regulatory matrix W, μ and Θ are parameters. The results on the structure and properties of attractive sets are provided. For the particular case of W being the matrix with unity entries everywhere except the main diagonal, where entries are zero, full description of attracting sets is given for any possible value of (μ, Θ).

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EXPONENTIAL STABILITY OF LINEAR DISCRETE SYSTEMS WITH MULTIPLE DELAYS

JOSEF DIBLÍK

Brno University of Technology

Technická 10, 616 00 Brno, Czech Republic

E-mail: `diblik@feec.vutbr.cz`

Recently, growing interest is paid to the investigation of stability of linear difference systems with delay. In the talk we give sufficient conditions for the exponential stability of linear difference systems with multiple delays

$$x\left(k+1\right)=Ax\left(k\right)+\sum_{i=1}^{s}B_i(k)x\left(k-m_i(k)\right), \quad k=0,1,\dots \tag{1}$$

where A is an $n\times n$ constant matrix, $B_i(k)$ are $n\times n$ matrices, $m_i(k)\in\mathbb{N}$, $m_i(k)\le m$ for an $m\in\mathbb{N}$, $s\in\mathbb{N}$ and $x=(x_1,\dots,x_n)^T\colon\{-m,-m+1,\dots\}\to\mathbb{R}^n$. Simultaneously, we give an exponential estimate of the norms of solutions. The results are compared with some previously published results. The exponential stability of (1) is studied by the second Lyapunov method. Investigation is performed by Lyapunov function $V(x)=x^THx$ with an $n\times n$ positive definite symmetric matrix H.

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Abstracts of PODE 2016, May 17–20, 2016, Rīga, Latvia

DISCRETE PAINLEVÉ EQUATIONS FOR RECURRENCE COEFFICIENTS OF LAGUERRE-HAHN ORTHOGONAL POLYNOMIALS OF CLASS ONE

GALINA FILIPUK

University of Warsaw, IMPAN, KU Eichsstaett-Ingolstadt

Ostenstr. 26-28 Eichstaett, D-85071, Germany

E-mail: filipuk@mimuw.edu.pl

We study recurrences for Laguerre-Hahn orthogonal polynomials of class one. It is shown for some families of such Laguerre-Hahn polynomials that the coefficients of the three term recurrence relation satisfy some forms of discrete Painlevé equations, namely, dP_I and dP_{IV}. This is a joint work [1] with M.N. Rebocho (Departmento de Matemática, Universidade da Beira Interior, CMUC, Coimbra, Portugal).

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ON THE FACTORIZATION OF CERTAIN DIFFERENCE EQUATIONS

GALINA FILIPUK

KU Eichstaett-Ingolstadt

Ostenstr. 26-28 Eichstaett, Germany

E-mail: filipuk@mimuw.edu.pl

In this talk I shall present some recent results on the factorization of certain (q, h)–difference equations. The talk will be based on the papers [1; 2].

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ON GLOBAL BIFURCATIONS OF LIMIT CYCLES IN CONTINUOUS AND DISCRETE POLYNOMIAL DYNAMICAL SYSTEMS

VALERY GAIKO

National Academy of Sciences of Belarus
United Institute of Informatics Problems

L. Beda Str. 6, Minsk 220040, Belarus

E-mail: `valery.gaiko@gmail.com`

The global qualitative analysis of continuous and discrete polynomial dynamical systems is carried out [1]. First, using new bifurcational and topological methods, we solve *Hilbert's Sixteenth Problem* on the maximum number of limit cycles and their distribution for the 2D general Liénard polynomial system [2] and Holling-type quartic dynamical system [3]. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles which is related to *Smale's Fourteenth Problem* [4]. We discuss also how to apply our approach for studying global limit cycle bifurcations of discrete polynomial dynamical systems which model the population dynamics in biomedical and ecological systems.

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FLOWS OF HOMOGRAPHIES

DOROTA GŁAZOWSKA

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra

prof. Z. Szafrana 4a, 65-516 Zielona Góra, POLAND

E-mail: `D.Glazowska@wmie.uz.zgora.pl`

If the difference of two real homographic functions is nonnegative, then it is constant. Motivated by this property we introduce the following

Definition 1. Let φ and ψ be real homographic functions. We say that the pair (φ, ψ) is subcommuting (or φ subcommutes with ψ) if

$$\varphi \circ \psi \leq \psi \circ \varphi.$$

If the opposite inequality holds we say that the pair (φ, ψ) is supercommuting (or φ supercommutes with ψ).

We determine all pairs of subcommuting (supercommuting) real homographic functions. We also show that simple modification of subcommuting (supercommuting) functions transforms them into commuting ones. One of main results reads as follows

Theorem 2. *Assume that the homographic functions φ and ψ are of the form*

$$\varphi(x) = ax + b \qquad \text{and} \qquad \psi(x) = \frac{px + q}{x + r},$$

where $a, b, p, q, r \in \mathbb{R}$ and $a \neq 0$, $(a-1)^2 + b^2 \neq 0$, $pr \neq q$. The pair (φ, ψ) is subcommuting if and only if

$$a = -1, \qquad r = -b/2, \qquad p \geq b/2. \tag{1}$$

Moreover, if condition (1) holds and $p \neq \frac{b}{2}$, then the homographic functions f and g given by

$$f(x) = \varphi(x) + m, \qquad g(x) = \psi(x) + n,$$

for some $m, n \in \mathbb{R}$, are commuting if and only if $m = 0$ and $n = \frac{b-2p}{2}$.

Moreover we deal with one parameter families of comparable commuting homographic functions. In particular, we show that a generalized flow of comparable homographic functions coincides with the family of translations of the identity function.

Results have been obtained jointly with J. Matkowski.

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OSCILLATORY PROPERTIES OF SOLUTIONS OF DIFFERENCE EQUATIONS WITH DEVIATING ARGUMENTS

ALINA GLESKA and MAŁGORZATA MIGDA

Institute of Mathematics, Poznan University of Technology

Piotrowo 3a, PL60-965 Poznań, Poland

E-mail: alina.gleska@put.poznan.pl, malgorzata.migda@put.poznan.pl

In this talk we present some new criteria for the oscillation of solutions of the difference equation with retarded arguments

$$(E_z) \qquad (-1)^z \Delta^m x(n) = f(n, x(\sigma_1(n)), x(\sigma_2(n)), \dots, x(\sigma_k(n))),$$

where $z,\ k \in N,\ \ m \geq 2,\ n \in N_{n_0},\ \sigma_i : N_{n_0} \to N_{n_0}$ are functions such that $\lim_{n\to\infty} \sigma_i(n) = \infty$ $(i = 1, 2, \dots, k)$ and the function $f : N_{n_0} \times R^k \to R$ satisfies the conditions

$$(C_1) \qquad x_1 f(n, x_1, x_2, \dots, x_k) > 0 \quad \text{for} \quad x_1 x_i > 0 \quad (i = 1, 2, \dots, k)$$

and

$$(C_2) \qquad f(n, x_1, x_2, \dots, x_k)\ sgn\ x_1 \geq \sum_{i=1}^{k} p_i(n)|x_i|,$$

where $n \in N_{n_0},\ \ p_i : N_{n_0} \to R_+ \cup \{0\},\ \ (i = 1, 2, \dots, k)$.

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A NON-AUTONOMOUS GENERALIZATION OF THE $Q_{\rm V}$ EQUATION

GIORGIO GUBBIOTTI

Dipartimento di Matematica e Fisica and INFN Sezione Roma Tre

Via della Vasca Navale 84, 00146, Roma, Italy

E-mail: `gubbiotti@mat.uniroma3.it`

At the beginning of this century Adler, Bobenko and Suris started a program of classification of non-linear lattice equation depending on two discrete indices $n, m \in \mathbb{Z}$ based on the so-called *Consistency Around the Cube (CAC)*. Interest in such procedure was motivated from the fact that CAC provides a Lax pair, making the equations *integrable*. The main result of this program is the so called ABS list of lattice equations [1]. Few years later it was proved [8] that the equations of the ABS list were all particular cases of a single integrable equation: the $Q_{\rm V}$ equation

$$\begin{aligned} Q_{\rm V}:\ & a_1 u_{n,m} u_{n+1,m} u_{n,m+1} u_{n+1,m+1} \\ & + a_2 \left(u_{n,m} u_{n,m+1} u_{n+1,m+1} + u_{n+1,m} u_{n,m+1} u_{n+1,m+1} + u_{n,m} u_{n+1,m} u_{n+1,m+1} + u_{n,m} u_{n+1,m} u_{n,m+1}\right) \\ & + a_3 \left(u_{n,m} u_{n+1,m} + u_{n,m+1} u_{n+1,m+1}\right) + a_4 \left(u_{n,m} u_{n+1,m+1} + u_{n+1,m} u_{n,m+1}\right) \\ & + a_5 \left(u_{n+1,m} u_{n+1,m+1} + u_{n,m} u_{n,m+1}\right) + a_6 \left(u_{n,m} + u_{n+1,m} + u_{n,m+1} + u_{n+1,m+1}\right) + a_7 = 0, \end{aligned}$$

where the a_i are 7 arbitrary coefficients.

R. Boll [2; 3; 4] extended the results of ABS giving a list of non autonomous equations.

In this lecture we introduce a non-autonomous generalization of the $Q_{\rm V}$ equation which contains all the equations presented by Boll. Using the Algebraic Entropy test [5; 6; 7] we infer that such equation should be integrable and with the aid of a formula introduced by Xenitidis [9] we find its three point generalized symmetries.

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FRACTIONAL ITERATES OF PIECEWISE MONOTONIC FUNCTIONS OF NONMONOTONICITY HEIGHT NOT LESS THAN 2 – PART I

WITOLD JARCZYK

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra

Szafrana 4a, Pl-65-516 Zielona Góra, Poland

E-mail: `w.jarczyk@wmie.uz.zgora.pl`

This is a joint work with Liu Liu, Lin Li, and Weinian Zhang.

Given a set X, a self mapping F of X and a positive integer n a function $f : X \to X$ is said to be a *fractional iterate* (or: *iterative root*) *of order* n *of* F if

$$f^n(x) = F(x), \quad x \in X,$$

where f^n denotes the nth iterate of f. The problem of finding fractional iterates started from the papers [1] and [2] by Ch. Babbage published two hundred years ago. Some classical results were given by Bödewadt [3], and Kuczma [4].

A big part of the research was devoted to roots of monotonic functions. In 1961, in the paper [4], Kuczma gave a complete description of iterative roots of continuous strictly monotonic self-mappings of a given interval (see also the books [5] and [6]). In the talk we deal with fractional iterates of continuous piecewise monotonic functions defined on a compact interval.

Let $a, b \in \mathbb{R}, a < b$, and let $F : [a, b] \to \mathbb{R}$ be a continuous function. A point c of the interval (a, b) is called a *fort of* F if F is strictly monotonic in no neighbourhood of c. The function F is said to be *piecewise monotonic* if it has only finitely many forts. The set of all piecewise monotonic self-mappings of $[a, b]$ is denoted by $\mathcal{PM}(a, b)$. Given any $F \in \mathcal{PM}(a, b)$ we denote by $S(F)$ the set of all forts of F. It is known that $S\left(F^k\right) \subset S\left(F^{k+1}\right)$, $k \in \mathbb{N}$. If there is a $k \in \mathbb{N}$ such that $S\left(F^k\right) = S\left(F^{k+1}\right)$, then the least one is called the *nonmonotonicity height of* F and denoted by $H(F)$. Otherwise we put $H(F) = +\infty$.

In the talk we focus on functions $F \in \mathcal{PM}(a, b)$ satisfying $H(F) \geq 2$. We are interested in their roots of order $n = \#S(F)$. Assume that $n \geq 2$ and write $S(F) = \{c_1, \ldots, c_n\}$, where $a < c_1 < \ldots < c_n < b$. Set also $c_0 = a$ and $c_{n+1} = b$.

Lemma. *Let $F \in \mathcal{PM}(a, b)$ with $H(F) \geq 2$ and assume that $\#S(F) \geq 2$. If $f \in \mathcal{PM}(a, b)$ is a root of order $n\#S(F)$ of F, then f has exactly one fort. Moreover, one the following cases holds:*

$\mathcal{T}_1^-$. *$S(f) = \{c_1\}$, the function f reaches the minimum value at c_1 and $f(c_1) < c_1$,*

$\mathcal{T}_1^+$. *$S(f) = \{c_n\}$, the function f reaches the maximum value at c_n and $f(c_n) > c_n$,*

$\mathcal{T}_2^-$. *$S(f) = \{c_1\}$, the function f reaches the maximum value at c_1 and $f(c_1) > c_1$,*

$\mathcal{T}_2^+$. *$S(f) = \{c_n\}$, the function f reaches the minimum value at c_n and $f(c_n) < c_n$.*

A full characterization of those $F \in \mathcal{PM}(a, b)$ that have a root of order $n = \#S(F)$ of types $\mathcal{T}_1^-$ and $\mathcal{T}_1^+$ was given in [7]. We present it during the talk as well as a construction of all such roots.

For the references see those given at the end of the next talk being a continuation of this one.

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FRACTIONAL ITERATES OF PIECEWISE MONOTONIC FUNCTIONS OF NONMONOTONICITY HEIGHT NOT LESS THAN 2 – PART II

JUSTYNA JARCZYK

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra

Szafrana 4a, Pl-65-516 Zielona Góra, Poland

E-mail: `j.jarczyk@wmie.uz.zgora.pl`

This is a continuation of the previous talk. I use the same notation and definitions. The results that I am going to present were obtained jointly with Witold Jarczyk, Liu Liu and Weinian Zhang.

I consider the remaining cases $\mathcal{T}_2^-$ and $\mathcal{T}_2^+$ which surprisingly are much more complicated than $\mathcal{T}_1^-$ and $\mathcal{T}_1^+$ studied in [7]. The main theorem presented in my talk describes completely the situation; the problem has been solved by considering eight complementary cases (cf. [8]).

Among tools which are basic in the proof there are following two lemmas.

Lemma 1. *Let $F \in \mathcal{PM}(a,b)$ with $H(F) \geq 2$ and assume that $\#S(F) \geq 2$. If $f \in \mathcal{PM}(a,b)$ is a root of F, of order $n = \#S(F)$ and type $\mathcal{T}_2^-$, then*

$$f(c_i) = c_{n+2-i}, \quad i \in \left\{2, \ldots, \left[\frac{n+1}{2}\right]\right\}, \qquad \textit{and} \qquad f(c_i) = c_{n+1-i}, \quad i \in \left\{\left[\frac{n+1}{2}\right]+1, \ldots, n\right\}.$$

In the next lemma we use the following notion of *compatibility*. Continuous strictly increasing functions $F : [a,b] \to [a,b]$ and $G : [c,d] \to [c,d]$ are said to be *compatible* if $a \in \mathrm{Fix}F$ iff $c \in \mathrm{Fix}G$ and $b \in \mathrm{Fix}F$ iff $d \in \mathrm{Fix}G$ and there is a continuous strictly increasing function γ mapping $\mathrm{Fix}F$ onto $\mathrm{Fix}G$ such that $F(x)-x$ and $G(y)-y$ have the same signs in the intervals (ξ_1, ξ_2) and $(\gamma(\xi_1), \gamma(\xi_2))$, respectively, for every $\xi_1, \xi_2 \in \mathrm{Fix}F$ with $\xi_1 < \xi_2$ and $(\xi_1, \xi_2) \cap \mathrm{Fix}F = \emptyset$.

Lemma 2. *Let I and J be compact intervals and let $F : I \to I$ and $G : J \to J$ be compatible functions. Then, for any even $n \in \mathbb{N}$, there exist a continuous strictly increasing function $f : I \to J$ and continuous strictly decreasing function $g : J \to I$ such that $(g \circ f)^n = F$ and $(f \circ g)^n = G$.*

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SET INVARIANCE FOR DELAY DIFFERENCE EQUATIONS

MOHAMMED-TAHAR LARABA, SORIN OLARU and SILVIU-IULIAN NICULESCU

Laboratory of Signals and Systems (L2S)

CentraleSupeléc-CNRS-Université Paris-Saclay, France

E-mail: `mohammed.laraba@lss.supelec.fr`

In this talk, set invariance properties for dynamical systems described by linear discrete time-delay difference equations (dDDEs) of the form:

$$x(k+1) = \sum_{i=0}^{d} A_i x(k-i) \tag{1}$$

are addressed, where $x(k) \in \mathbb{R}^n$ is the state vector at the time $k \in \mathbb{Z}_+$, $d \in \mathbb{Z}_+$ is the maximal *fixed* time-delay, the matrices $A_i \in \mathbb{R}^{n \times n}$, for $i \in \mathbb{Z}_{[0,d]}$ and the initial conditions are considered to be given by $x(-i) = x_{-i} \in \mathbb{R}^n$, for $i \in \mathbb{Z}_{[0,d]}$. An extended state space representation can be constructed for any given delay realization. Just by setting $z(k+1) = \left[x(k+1)^T \cdots x(k-d+1)^T\right]^T$, equation (1) can be rewritten as:

$$z_{k+1} = A_e z_k = \begin{bmatrix} A_0 & \dots & A_{d-1} & A_d \\ I & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & I & 0 \end{bmatrix} z_k \tag{2}$$

The first goal is to review known necessary and/or sufficient conditions for the existence of invariant sets with respect to (1) and (2). Secondly, I will discuss recent results related to the invariance with respect to (1), also called $\mathcal{D}$-invariance. Set invariance in the original state space leads to conservative definitions due to its delay independent property. This limitation makes the $\mathcal{D}$-invariant sets only applicable to a limited class of systems. Hence an alternative solution based on the set factorization is established in order to obtain more flexible set characterization. With linear algebra manipulations it is shown that similarity transformations are key elements in the characterization of low complexity invariant sets. In short, it is shown that we can construct, in a low dimensional state-space, an invariant set for a dynamical system governed by a delay difference equation. The artifact which enables the construction is a simple change of coordinates for the dDDE (1). Interestingly, this $\mathcal{D}$-invariant set will be shown to exist in the new coordinates even if in its original state space it does not fulfill the necessary conditions for the existence of $\mathcal{D}$-invariant sets. This proves the importance of the state representation's choice.

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A DYNAMIC APPROACH TO CONSTANT ESCAPEMENT HARVESTING

EDUARDO LIZ

Departamento de Matemática Aplicada II, Universidade de Vigo

E. I. Telecomunicación, Campus Universitario, 36310 Vigo (Spain)

E-mail: eliz@dma.uvigo.es

Optimal harvesting strategies from an ecological perspective aim to promote stability and persistence of populations, thus preventing the risk of collapses. It has been suggested that a strategy of constant escapement (also referred to as *threshold harvesting*) is optimal for sustainable development [3; 4]. Such an strategy consists of harvesting all individuals above a threshold population size, with no harvest below the threshold.

For discrete-time single-species populations, we can use a simple one-dimensional map to analyze the dynamical properties of this harvesting strategy, namely,

$$x_{n+1} = \min\{f(x_n), T\} = \begin{cases} f(x_n) & \text{if } f(x_n) \le T, \\ T & \text{if } f(x_n) > T, \end{cases} \tag{1}$$

where x_n is the population size at the nth generation (after harvesting), $T > 0$ is the threshold, and f is the recruitment function. The flat-topped map defined by the right-hand side of equation (1) has also been considered in the context of chaos control [2].

We describe the typical dynamics of the solutions of (1) for the usual compensation and depensation models [1], and we compare the method with other usual management strategies (constant effort and constant yield). Our results confirm that constant escapement favors stability and persistence in contrast with the other methods.

This talk is based on a joint work with Prof. Frank Hilker.

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A TIME SCALES APPROACH TO THE COMPETITIVE EXCLUSSION PRINCIPLE

MARCOS MARVÁ

Departamento de física y matemáticas. Universidad de Alcalá

Alcalá de Henares, 28871, Spain

E-mail: marcos.marva@uah.es

The competitive exclusion principle is a tenet in theoretical ecology and its validity was tested in the well known experiments with flour beetle [1, 2]. It is known that the data observed in these experiments mainly corroborated the exclusion principle, whose outcomes are captured by the Leslie-Gower model, the discrete counterpart of the Lotka-Volterra competition model [3].

However, part of the data was at odds with this model. Subsequently, Edmunds, Cushing and collaborators [4] showed that the celebrated PLA model (larva-pupae-adult), a competition model that incorporates age structure, may exhibit long term behaviour compatible with this unexpected data.

In this work, and based on behavioral features of the species of the flour beetle involved in the experiment, we present an alternative approach that also explains the aforementioned unexpected behavious. In this case, the model relies on spatial heterogeneity and fast individuals dispersal [5] rather that on age structure.

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GEOGEBRA: A SUITABLE INTERACTIVE TOOL FOR EXPLORING DISCRETE DYNAMICAL SYSTEMS

MARCOS MARVÁ and FERNANDO SAN SEGUNDO

Departamento de física y matemáticas. Universidad de Alcalá

Alcalá de Henares, 28871, Spain

E-mail: `marcos.marva@uah.es`

GeoGebra [1] is free software that allows to interact with mathematical entities from both the geometrical and the algebraic point of view. It was originally designed for teaching purposes with emphasis in manipulability and interactivity, and these features make it simple to generate, visualize and explore the behavior of solutions and orbits of low dimensional discrete systems.

In particular, given a difference equations system, it is straightforward to parametrize the coefficients of the system by means of sliders, in such a way that the corresponding cobweb diagram/solutions/orbits of the system are sensitive, in real time, to variations on the parameters values by just dragging the slider with the mouse.

We use these tools for both teaching and preliminary research purposes. Students and us benefit from the chance of interactively explore the behaviour of the system for a large range of parameters values and, thus, gaining intuition on the features of the underlaying model.

This poster displays examples of what can be done, and provides links to a GitHub [2, 3] that stores

- to reproducible documents, a kind of tutorials, that describe how to build up the dynamical system. Suitable for students and researchers
- applet examples that can be easily customized.

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BOUNDEDNESS OF SOLUTIONS OF DYNAMIC EQUATIONS ON TIME SCALES

JAQUELINE MESQUITA

Universidade de Brasília

Departamento de Matemática, Campus Universitário Darcy Ribeiro, Brasília-DF, Brazil

E-mail: jgmesquita@unb.br

This is a joint work with Professors Márcia Federson, Rogélio Grau and Eduard Toon. The goal of this paper is to study the boundedness of the solutions of dynamic equations on time scales using Lyapunov functionals, considering more general conditions. In order to obtain our results, we investigate the boundedness results for measure differential equations and we use the correspondence between these equations and the dynamic equations on time scales to extend the results for these last equations.

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COMPARISON THEOREMS FOR THIRD-ORDER DELAY TRINOMIAL DIFFERENCE EQUATIONS

MAŁGORZATA MIGDA and ALINA GLESKA

Institute of Mathematics, Poznań University of Technology

Piotrowo 3A, 60-965 Poznań, Poland

E-mail: malgorzata.migda@put.poznan.pl,alina.gleska@put.poznan.pl

We consider the third-order delay trinomial difference equation of the form

$$\Delta^3 x_n + p_n \Delta x_{n+1} + q_n x_{n-\tau} = 0,$$

where τ is a positive integer, (p_n) is a sequence of nonnegative real numbers, (q_n) is a sequence of positive real numbers. We transform this equation to a binomial third-order difference equation with quasidifferences. Using comparison theorems with a certain first order delay difference equation we establish results on some asymptotic properties of solutions of the studied equation. The presented criteria is easily applicable.

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A FAST ALGORITHM FOR NUMERICAL SOLVING OF SPECIAL TYPE EQUATION

RAITIS OZOLS

University of Latvia

Raiņa bulvāris 19, Rīga LV-1586, Latvia

E-mail: raitis.ozols@inbox.lv

The equation

$$f(x) := \frac{a_1}{x^{b_1}} + \frac{a_2}{x^{b_2}} + \ldots + \frac{a_n}{x^{b_n}} = C,$$

where $n \geq 2$, $C > 0$, $0 < b_1 < b_2 < \ldots < b_n$, and $a_1, a_2, \ \ldots, a_n > 0$, appearing in some banks' calculations of annual percentage rate is considered. The attention is paid to the problem of finding the unique real positive root of this equation with high accuracy using as few as possible number of mathematical operations.

The proposed fast root-finding algorithm is the following iterative method

$$x_1 = \left(\frac{a_1 + a_2 + \ldots + a_n}{C}\right)^{n/(b_1+b_2+\ldots+b_n)}, \ x_{k+1} = x_k \cdot \left(\frac{f(x_k)}{C}\right)^{-\frac{1}{x_k} \cdot \frac{f(x_k)}{f'(x_k)}}, \ k \geq 1.$$

Practical testing shows that a few iterations are enough (so x_5 is very close to the unique positive root). It seems that $\frac{1}{|x_0 - x_n|}$ always grows as fast as A^{n^B} where x_0 is the positive root of equation and A and B are constants, greater than 1.

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ABOUT NEURON MODEL WITH PERIOD THREE INTERNAL DECAY RATE

MICHAEL A. RADIN[1], INESE BULA[2,3] and NICHOLAS WILKINS[1]

[1] *Rochester Institute of Technology*

School of Mathematical Sciences, Rochester, New York 14623, U.S.A.

[2] *Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 25, Rīga LV-1002, Latvia

[3] *Institute of Mathematics and Computer Science, University of Latvia*

Raiņa bulvāris 29, Rīga LV-1459, Latvia

E-mail: michael.radin@rit.edu, ibula@lanet.lv, npw3202@rit.edu

In [1] a difference equation $x_{n+1} = \beta x_n - g(x_n), \quad n = 0, 1, 2, ...$, was analyzed as a single neuron model, where $\beta > 0$ is an internal decay rate and a signal function g is the following piecewise constant function with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{1}$$

Now we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, ...,$$

where $(\beta_n)_{n=0}^{\infty}$ is a period three sequence

$$\beta_n = \begin{cases} \beta_0, & \text{if } n = 3k, \\ \beta_1, & \text{if } n = 3k+1, \quad k = 0, 1, 2, ... \\ \beta_2, & \text{if } n = 3k+2, \end{cases}$$

and g is in form (1).

In [2] we have been studied this model where $(\beta_n)_{n=0}^{\infty}$ is a period two sequence. The goal of this paper is to investigate the boundedness nature and the periodic character of solutions. Furthermore, we will determine the relationships of the periodic cycles relative to the periods of the parameters and relative to the relationship between the parameters as well. Moreover, we will investigate which particular periodic cycles can be only periodic and which particular periodic solutions can be eventually periodic. In addition, we will show the bifurcation diagrams when solutions transition from periodicity of various periods to unbounded solutions.

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NODAL TYPE NUMERICAL METHOD FOR 2D ABSORPTION EQUATION WITH DIRICHLET BOUNDARY CONDITIONS

MICHAEL A. RADIN[1], PATRIKS MOREVS[2] and MAXIMS ZIGUNOVS[3]

[1] *Rochester Institute of Technology*
School of Mathematical Sciences, Rochester, New York 14623, U.S.A.
[2] *Foreign Affairs Department, Liepaja University*
Liela Iela 14, Liepaja LV-3410, Latvia
[2] *Faculty of Science and Engineering, Liepaja University*
Liela Iela 14, Liepaja LV-3410, Latvia
[3] *Faculty of Science and Engineering, Liepaja University*
Liela Iela 14, Liepaja LV-3410, Latvia
E-mail: `michael.radin@rit.edu, acentrs@liepu.lv, maksims.zigunovs@inbox.lv`

In [2], the authors studied the Construction and Analysis of the following functional modal type difference scheme for 2D Helmholtz Equation:

$$G_x D_x U_{xx}(x,y) \;+\; G_y D_y U_{yy}(x,y) \;+\; P(x,y) * U(x,y) \;=\; 0\;.$$

The goal of this paper is to study the Nodal Type Numerical Method for 2D Absorption Equation with Dirichlet Boundary Conditions. Furthermore, to compare the differences with the speed of computation, with the precision, with the A.D.I. method and with Comsol.

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BOHL-PERRON PRINCIPLE FOR DYNAMIC EQUATIONS ON TIME SCALES*

ANDREJS REINFELDS[1,2] and DZINTRA ŠTEINBERGA[2]

[1] *Institute of Mathematics and Computer Science*
Raiņa bulvāris 29, Rīga LV-1459, Latvia
[2] *University of Latvia, Department of Mathematics*
Zeļļu iela 25, Rīga LV-1002, Latvia
E-mail: `reinf@latnet.lv, dzintra.steinberga@gmail.com`

We consider the dynamic equation in a Banach space on unbounded above and below time scales $\mathbb{T}$:

$$x^{\Delta} \;=\; A(t)x + f(t,x), \tag{1}$$

with rd-continuous, regressive right hand side, nonlinear term satisfy the Lipschitz condition

$$|f(t,x) - f(t,x')| \le \varepsilon(t)|x - x'|,$$

and the estimate

$$\sup_x |f(t,x)| \le N(t) < +\infty.$$

where $N\colon \mathbb{T} \to \mathbb{R}_+$ and $\varepsilon\colon \mathbb{T} \to \mathbb{R}_+$ are integrable scalar functions.

Using Green type mapping [1] we find sufficient condition for the existence of bounded solution and investigate it's properties.

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DEMAND-INVESTORY MODEL

EWA SCHMEIDEL

Institute of Mathematics

ul. Ciołkowskiego 1M, 15-245 Białystok, Poland

E-mail: eschmeidel@math.uwb.edu.pl

Prediction of future demand and inventory is an important aspect of running and managing manufacturing or trade company. Methods supporting those tasks have been developed by economists already in the mid of twentieth century, nonetheless they are still being improved as economy still changes and creates new challenges.

Ma and Feng in [3] proposed the dynamical model of demand and inventory with mechanism of demand stimulation and inventory limitation. The model describes demand and inventory of a product at one echelon of supply chain - at retailer. Considered supply chain consists of three echelons: manufacturer, retailer and customers. Following rules are applied to the model: customers buy a good from a retailer, a retailer orders a product in the forecasted amount and forecast is prepared using single exponential smoothing model of Brown ([1]), a manufacturer produces and delivers exactly ordered amount and production capacity is unlimited, customers' demand depends on a retail price, which can be changed by a discount, price cannot be arbitrary changed but the retailer can offer a discount depending on stock volume: when stock is high, the retailer offers a discount to encourage customers to buy a product. The model takes a form of the following system of difference equations

$$\begin{cases} D_{n+1} &= \left[\dfrac{AT}{(A+1)T - S_n}\right]^k D_n \\ S_{n+1} &= S_n - D_n + \check{D}_n \\ \check{D}_{n+1} &= \alpha D_n + (1-\alpha)\check{D}_n \end{cases} \tag{1}$$

where: $n \in \mathbb{N}$, S_n is a stock volume, $D_n \geq 0$ is a demand volume, $\check{D}_n \geq 0$ is a forecast of demand at n and ordered placed by a retailer at a manufacturer, moreover by assumption of unlimited capacity it is also delivered quantity at n, $A > 0$ is a parameter for discount steering, $T > 0$ is a parameter for defining the target stock, $k > 0$ is price elasticity coefficient that regulates dependence between price, discount and demand, $\alpha \in (0, 1)$ is a forecast smoothing coefficient.

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PERSISTENCE AND EXTINCTION IN DISCRETE NON-AUTONOMOUS EPIDEMIC MODELS WITH GENERAL INCIDENCE

CÉSAR M. SILVA

Universidade da Beira Interior

Rua Marquês d'Ávila e Bolama, 6201-001 Covilhã, Portugal

E-mail: `csilva@ubi.pt`

In [1; 2; 3] three families of non-autonomous epidemic models with general incidence rate given by general functions were considered and threshold conditions for the persistence and extinction of the disease were obtained. It was also established that in the case of extinction, we have global asymptotic stability. The particular autonomous and periodic settings as well as particular forms for the incidence function were also discussed.

The objective of this talk is to obtain corresponding results for discrete-time versions the models in [1; 2; 3], that were derived by applying Mickens nonstandard finite difference method [4] to the corresponding continuous models. Additionally, we discuss the dependence of the thresholds on the incidence functions.

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WELL-POSEDNESS AND MAXIMUM PRINCIPLES FOR LATTICE REACTION-DIFFUSION EQUATIONS

ANTONÍN SLAVÍK

Charles University in Prague, Faculty of Mathematics and Physics

Sokolovská 83, 186 75 Praha 8, Czech Republic

E-mail: `slavik@karlin.mff.cuni.cz`

The classical reaction-diffusion equation $\partial_t u(x,t) = k\partial_{xx}u(x,t) + f(u(x,t))$ describes the evolution of chemical concentrations, temperatures, or populations. These phenomena combine a local dynamics (via the reaction function f) and a spatial dynamics (via the diffusion).

Motivated by applications in biology and chemistry, various authors have considered the lattice reaction-diffusion equation

$$\partial_t u(x,t) = k(u(x+1,t) - 2u(x,t) + u(x-1,t)) + f(u(x,t)), \quad x \in \mathbb{Z}, \quad t \in [0,\infty), \tag{1}$$

as well as the discrete reaction-diffusion equation

$$u(x,t+1) - u(x,t) = k(u(x+1,t) - 2u(x,t) + u(x-1,t)) + f(u(x,t)), \quad x \in \mathbb{Z}, \quad t \in \mathbb{N}_0. \tag{2}$$

Equations (1) and (2) are also interesting from the standpoint of numerical mathematics, since they correspond to semi- or full discretization of the original reaction-diffusion equation.

In order to study both (1) and (2) in a unified way, we use the language of the time scale calculus and consider the nonautonomous lattice reaction-diffusion equation

$$u^{\Delta}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t) + f(u(x,t),x,t), \quad x \in \mathbb{Z}, \quad t \in \mathbb{T}, \tag{3}$$

where $a,b,c \in \mathbb{R}$, $\mathbb{T} \subseteq \mathbb{R}$ is a time scale, and u^{Δ} denotes the Δ-derivative with respect to time.

Our results are new even in the special cases $\mathbb{T} = \mathbb{R}$ (when $u^{\Delta}(x,t)$ becomes the partial derivative $\partial_t u(x,t)$) and $\mathbb{T} = \mathbb{Z}$ (when $u^{\Delta}(x,t)$ is the partial difference $u(x,t+1) - u(x,t)$). First, we focus on the local existence and global uniqueness of bounded solutions, as well as continuous dependence of solutions on the underlying time scale and on initial conditions. The proofs are based on reformulating the reaction-diffusion equation as an abstract dynamic equation, and also on techniques from the Kurzweil-Stieltjes integration theory. Next, we obtain the weak maximum principle, which enables us to get global existence of solutions. Finally, we provide the strong maximum principle, which exhibits an interesting dependence on the time structure.

Special cases of equation (3) include the autonomous Fisher and Nagumo lattice equations, or nonautonomous logistic population models with a variable carrying capacity.

This talk is based on a joint paper with Petr Stehlík and Jonáš Volek (University of West Bohemia, Czech Republic).

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REACTION-DIFFUSION EQUATIONS ON GRAPHS.

PETR STEHLIK

Department of Mathematics and NTIS

Technicka 8, 30614 Pilsen, Czech Republic

E-mail: `pstehlik@kma.zcu.cz`

In this talk, we deal with discrete- and continuous-time reaction-diffusion equations on general connected undirected graphs. We briefly discuss our motivation which arises from simple biological and economic models. First, we discuss the role of the time structure, especially on existence of solutions and a priori estimate. Next, we reveal that the rich spatial structure (graphs) give rise to dynamical properties which are not present in standard models (e.g., asymptotically stable coexistence stationary solutions).

This talk is based on the joint papers with Antonín Slavík (Charles University, Prague) and Jonáš Volek (University of West Bohemia)

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ON SOME EXPONENTIAL-TYPE DIFFERENCE EQUATIONS

AGNESE SUSTE

University of Latvia, Faculty of Physics and Mathematics

Zellu Street 25, Riga, LV-1002, Latvia

E-mail: agnese.suste@lu.lv

In [2] authors proposed Research Project 6.7.1. and Research Project 6.7.2. about the difference equation

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)(1 - e^{-Ax_n}), \quad n = 0, 1, ..., \tag{1}$$

which is a special case of an epidemic model (see [1]).

In [3] authors proposed Open Problem 6.10.14 about the difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-Ax_n}), \quad n = 0, 1, ... \tag{2}$$

In [8] authors study the oscillation, global asymptotic stability, and other properties of positive solutions of the difference equation (1). In [6] authors investigate the global stability of the negative solutions of (1). In [5] authors considered the fuzzy difference equation (1). System of difference equations related to model (1) are studied in [4] and [7].

We investigate a difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-Ax_n - Bx_{n-1}}), \quad n = 0, 1, ... \tag{3}$$

where $A, B > 0$ and the initial values x_{-1}, x_0 are arbitrary real positive numbers such that $x_{-1} + x_0 < 1$.

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HYERS-ULAM STABILITY OF LINEAR DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS

A. K. TRIPATHY

Depatment of Mathematics, Sambalpur University

Sambalpur-768019, India

E-mail: arun_tripathy70@rediffmail.com

In this work, the Hyers-Ulam stability of linear difference equations of the form:

$$y_{n+1} - p_n y_n - r_n = 0$$

and

$$y_{n+2} + \alpha_n y_{n+1} + \beta_n y_n - r_n = 0$$

are studied, where p_n, α_n, β_n and r_n are the sequences of reals.
Keywords : Hyers-Ulam stability, difference equation.
Mathematics Subject Classification (2010) : 39A45, 39B42.

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POSITIVE SOLUTIONS FOR A SYSTEM OF DIFFERENCE EQUATIONS WITH COUPLED MULTI-POINT BOUNDARY CONDITIONS

RODICA LUCA TUDORACHE

Gh. Asachi Technical University

11 Blvd. Carol I, Iasi 700506, Romania

E-mail: rluca@math.tuiasi.ro

This is a joint work with Prof. Johnny Henderson (Baylor University, Waco, Texas, USA). We investigate the system of nonlinear second-order difference equations

$$\begin{cases} \Delta^2 u_{n-1} + \lambda f(n, u_n, v_n) = 0, \;\; n = 1, \ldots, N-1, \\ \Delta^2 v_{n-1} + \mu g(n, u_n, v_n) = 0, \;\; n = 1, \ldots, N-1, \end{cases} \tag{1}$$

with the coupled multi-point boundary conditions

$$u_0 = 0, \;\; u_N = \sum_{i=1}^{p} a_i v_{\xi_i}, \;\; v_0 = 0, \;\; v_N = \sum_{i=1}^{q} b_i u_{\eta_i}, \tag{2}$$

where $N \in \mathbf{N}$, $N \geq 2$, $p, q \in \mathbf{N}$, Δ is the forward difference operator with stepsize 1, $\Delta u_n = u_{n+1} - u_n$, $\Delta^2 u_{n-1} = u_{n+1} - 2u_n + u_{n-1}$, $a_i \in \mathbb{R}$, $\xi_i \in \mathbf{N}$ for all $i = 1, \ldots, p$, $b_i \in \mathbb{R}$, $\eta_i \in \mathbf{N}$ for all $i = 1, \ldots, q$, $1 \leq \xi_1 < \ldots < \xi_p \leq N-1$, $1 \leq \eta_1 < \cdots < \eta_q \leq N-1$, and λ and μ are positive parameters.

By using the Guo-Krasnosel'skii fixed point theorem, we present sufficient conditions on the parameters λ, μ and on the functions f, g such that positive solutions of (1) – (2) exist. The nonexistence of positive solutions for the above problem is also studied (see [1]). For some recent results on the existence and multiplicity of positive solutions for systems of difference equations with various uncoupled multi-point boundary conditions see the monograph [2].

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EXISTENCE AND UNIQUENESS FOR IMPLICIT DISCRETE NAGUMO EQUATION

JONÁŠ VOLEK

New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia in Pilsen

Univerzitní 8, 306 14 Plzeň

E-mail: volek1@kma.zcu.cz

This is a joint work with Petr Stehlík. We analyze existence and uniqueness of ℓ^2-solutions of the implicit discrete Nagumo reaction-diffusion equation. We study the infinite-dimensional problem variationally and describe corresponding potentials which have either the convex or mountain pass geometry. Consequently, we show that the implicit Nagumo equation has a solution for all reaction parameters $\lambda \in \mathbb{R}$, at least for small time discretization steps h. Moreover, the solution is unique in the bistable case, $\lambda > 0$.

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SOME PROPERTIES OF K-DIMENSIONAL SYSTEM OF NEUTRAL DIFFERENCE EQUATIONS

MAŁGORZATA ZDANOWICZ

Institute of Mathematics University of Bialystok

K. Ciołkowskiego 1M, 15-245 Białystok, Poland

E-mail: mzdan@math.uwb.edu.pl

The k-dimensional system of neutral type nonlinear difference equations with delays in the following form ($i = 1, \dots, k-1$)

$$\begin{cases} \Delta\Big(x_i(n) + p_i(n)\, x_i(n-\tau_i)\Big) = a_i(n)\, f_i(x_{i+1}(n-\sigma_i)) + g_i(n) \\ \Delta\Big(x_k(n) + p_k(n)\, x_k(n-\tau_k)\Big) = a_k(n)\, f_k(x_1(n-\sigma_k) + g_k(n) \end{cases}$$

is considered. The aim is to present sufficient conditions for the existence of nonoscillatory bounded solutions of the system above with various $[(p_1(n)), \dots, (p_k(n))]$.

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