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ON DIFFERENCE AND DISCRETE EQUATIONS

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We study the following equation

$$\sum_{|k|=0}^{+\infty} a_k(x)u(x+\alpha_k) = v(x), \quad x,\ \alpha_k \in D, \forall k, \tag{1}$$

where D is a convex cone in $\mathbf{R}^m$, k is a multi-index. There are many different situations related to the fact when x may be a continual or a discrete variable. For a continual variable $x \in D$ the function

$$\sigma(x,\xi) = \sum_{|k|=0}^{+\infty} a_k(x)e^{i\alpha_k \cdot \xi}, \quad \xi \in \mathbf{R}^m, \tag{2}$$

is called a symbol of the equation (1) if the series (2) converges $\forall x \in \overline{D},\ \xi \in \mathbf{R}^m$. We say that a symbol is called elliptic if it is non-vanishing for all possible x, ξ. We assume here that $\sigma(x,\xi) \in C(\dot{\mathbf{R}}^m \times \dot{\mathbf{R}}^m)$ (it is possible for example if $a_k(x)$ are continuos functions with compact supports, and the sum in (2) is finite).

Lemma 1. *If $D = \mathbf{R}^m$ and the symbol* (2) *does not vanish then the equation* (1) *has a Fredholm property in the space $L_2(\mathbf{R}^m)$.*

If $D = \mathbf{R}^m_+ \equiv \{x \in \mathbf{R}^m :\ x_m > 0\}$, then an ellipticity of the symbol $\sigma(x,\xi)$ is not enough.

Theorem 2. *Let $D = \mathbf{R}^m_+$. The equation* (1) *has a Fredholm property in the space $L_2(\mathbf{R}^m_+)$ iff the symbol $\sigma(x,\xi',\xi_m)$, $\xi = (\xi',\xi_m)$, is elliptic and*

$$\int_{-\infty}^{+\infty} d \arg \sigma(\cdot,\cdot,\xi_m) = 0.$$

For discrete equations similar results were described in [1; 2] using methods developed in [3].

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