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ON EXTREMAL SOLUTIONS OF SIXTH-ORDER BOUNDARY VALUE PROBLEMS

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For fixed $g \in L(I,[0,\infty))$, $\quad I=[0,1]$, the boundary value problem

$$x^{(n)} = f(t), \quad |f| \le g, \quad l_i x = 0, \quad i = 1,...,n, \tag{1}$$

is considered, where $f \in L(I,R)$, $\quad l_i x = x^{(m_i)}(0) \quad$ or $l_i x = x^{(m_i)}(1)$, $\quad 0 \le m_i \le n-1$, $\quad i = 1,...,n$. Let $a(m)=1$ for $m \in \{0,...,n-1\}$, if there exists $i \in \{1,...,n\}$ such that $l_i x = x^{(m)}(0)$, and let $a(m)=0$ otherwise. Likewise, $b(m)=1$ for $m \in \{0,...,n-1\}$, if there exists $i \in \{1,...,n\}$ such that $l_i x = x^{(m)}(1)$, and let $b(m)=0$ otherwise. We assume that

$$\sum_{k=0}^{m}(a(k)+b(k)) \ge m+1, \quad m \in \{0,...,n-2\}, \quad \sum_{k=0}^{n-1}(a(k)+b(k)) = n. \tag{2}$$

These conditions guarantee the existence and uniqueness of a solution x_f of the boundary value problem (1). Let X be a set of solutions of the boundary value problem (1) and let $y_m \in X$ for $m \in \{0,...,n-1\}$ be an extremal solution such that

$$\|y_m^{(m)}\|_C = max\{\|x^{(m)}\|_C : x \in X\}. \tag{3}$$

THEOREM 1. *If condition (2) holds for $n=6$, then the condition $y_m \in \{x_g, -x_g\}$, $\quad m \in \{0,1,...,5\}$, is satisfied for the boundary value problem (4)*

$$x^{(6)} = f(t), \quad |f| \le g, \quad l_i x = 0, \quad i = 1,...,6, \tag{4}$$

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