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ON THREE POINT BOUNDARY VALUE PROBLEM

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We consider the resonant three point boundary value problem

$$x'' + k^2 x = f(t, x), \tag{1}$$

$$x(0) = 0, \;\; x(1) = \delta x(\eta), \tag{2}$$

where $0 < \eta < 1$, $\delta > 0$, f may be unbounded [3].

The boundary value problem (1), (2) is called resonant if the respective homogeneous boundary value problem has nontrivial solutions. To get the existence of a solution to the problem (1), (2), we use the quasilinearization approach elaborated in the works [1], [2].

1. First modify the equation by adding a linear part so that the resulting linear part is not resonant yet
$$x'' + (k^2 + \varepsilon^2)x = \varepsilon^2 x + f(t, x) =: F(t, x), \tag{3}$$
where $\sin\sqrt{k^2+\varepsilon^2} - \delta \sin \eta\sqrt{k^2+\varepsilon^2} \neq 0$;
2. choose a constant $N > 0$ and truncate the right hand side
$$x'' + (k^2 + \varepsilon^2)x = F_N(t, x) := F(t, \delta(-N, x, N)); \tag{4}$$
3. check the inequality
$$\Gamma \cdot M \leq N, \tag{5}$$
where $\Gamma = \max\limits_{0 \leq t,s \leq 1} |G(t, s)|$ is the estimate of the Green's function associated with the linear part in (4) and boundary conditions (2), $M = \sup_{I \times R} |F_N(t, x)|$.

The original equation (1) and the modified equation (4) are equivalent in $[0, 1] \times [-N, N]$, therefore a solution $x(t)$ of the quasilinear boundary value problem (4), (2) is also a solution of the original problem (1), (2). The following Theorem 1 is proved.

THEOREM 1. *Suppose that ε^2 and N can be found such that the inequality (5) fulfils. Then the resonant problem (1), (2) has a solution $x(t)$, such that $\big|x(t)\big| \leq N$ for $t \in [0, 1]$.*

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