Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

ON THE STABILITY OF SOME THREE-LAYER DIFFERENCE SCHEMES FOR TWO-DIMENSIONAL PSEUDO-PARABOLIC EQUATION WITH INTEGRAL BOUNDARY CONDITIONS

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We consider the third order linear pseudo-parabolic equation in rectangle with nonlocal integral conditions

$$u_t = u_{xx} + u_{yy} + \eta(u_{xx} + u_{yy})_t + f(x, y, t), \quad (x, y) \in (0, L_1) \times (0, L_2), \tag{1}$$

$$u(0, y, t) = \gamma_1 \int_0^{L_1} u(x, y, t)\mathrm{d}x + \mu_1(t), \quad u(L_1, y, t) = \gamma_2 \int_0^{L_1} u(x, y, t)\mathrm{d}x + \mu_2(t), \tag{2}$$

$$u(x, 0, t) = \mu_3(x, t), \qquad u(x, L_2, t) = \mu_4(x, t), \tag{3}$$

where $\eta \geq 0$. Pseudo-parabolic equations with nonlocal boundary conditions arise from various physical phenomena, particulary, the dynamics of ground moisture. A very close mathematical model arises in the study of the incompressible non-newtonian flow problem. The stability of numerical methods was investigated in [1; 2].

We provide a numerical algorithm for the approximations of problem (1)–(3) based on the Peacemen–Rachford alternating direction implicit method[3]. According to this algorithm it is necessary to solve alternately two systems of difference equations with tridiagonal matrices, one of these systems is solved with nonlocal conditions. Each of systems is three-layer difference scheme ant it approximates the initial problem with truncation error $\mathcal{O}(\tau + h^2)$.

For investigation of stability of this method we rewrite the three-layer scheme in an equivalent form of a two-layer scheme $\mathbf{Y}^{n+1} = \mathbf{S}\mathbf{Y}^n$, where $\mathbf{Y}^n$ is vector defined into two layers, $\mathbf{S}$ is a nonsymmetric matrix. The stability conditions are derived in a specially defined energy norm by investigating of the spectrum of $\mathbf{S}$. For this end we analyse the auxillary nonlinear eigenvalue problem.

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[*] The research was partially supported by the Research Council of Lithuania (grant No. MIP-047/2014).