

INVESTIGATION OF A SPECTRUM FOR STURM–LIOUVILLE PROBLEM WITH PARTIAL INTEGRAL BOUNDARY CONDITION*

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Consider the Sturm–Liouville problem with one classical boundary condition

$$-u'' = \lambda u, \quad u(0) = 0, \quad t \in (0, 1),$$

and other integral boundary condition

$$u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t) dt,$$

with parameters $\gamma \in \mathbb{R}$, $\boldsymbol{\xi} = (\xi_1, \xi_2)$, $0 < \xi_1 < \xi_2 < 1$. In this work we investigate complex eigenvalues for special cases of $\boldsymbol{\xi}$ with rational components (the cases $\boldsymbol{\xi} = (0, 1)$, $\boldsymbol{\xi} = (1/4, 3/4)$, $\boldsymbol{\xi} = (\xi, 1)$ and $\boldsymbol{\xi} = (0, \xi)$ were investigated in [1; 2]). Our goal is to clarify an influence of parameters γ , ξ_1 , ξ_2 for the spectrum of Sturm–Liouville problem and observe a behavior of the critical points of Complex–Real Characteristic Function [3]. The investigation of critical points, connected with bifurcation points in space of parameter $\boldsymbol{\xi} = (\xi_1, \xi_2)$, is important for understanding the structure of eigenspectrum. Furthermore, the methods described in this work, allows to study the spectrum of more complex time depended models.

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*The research was partially supported by the Research Council of Lithuania (grant No. MIP-047/2014).