Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

STANDARD DIFFERENCE SCHEME IN THE PRESENCE OF PERTURBATIONS FOR A SINGULARLY PERTURBED PARABOLIC CONVECTION-DIFFUSION EQUATION

GRIGORII SHISHKIN

Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences

S. Kovalevskaya Str., 16, 620990, Yekaterinburg, Russia

E-mail: shishkin@imm.uran.ru

At present, only for sufficiently narrow class of singularly perturbed problems (problems with a perturbation parameter ε, $\varepsilon \in (0, 1]$, multiplying the highest-order derivative in the equation), special numerical methods were developed that converge ε-uniformly in the maximum norm (see., e.g., [1; 2] and the references therein). For this reason, well-developed numerical methods based on standard finite difference schemes (see., e.g., [3] and the references therein) are widely used in solving many scientific problems, and also they are often applied for solving problems with boundary layers. For convergence of solutions of such difference schemes, it is sufficient to use uniform grids provided that the mesh step-size across the boundary layer is much less than the value of the perturbation parameter ε [1; 2].

However, this simple recipe for solving singularly perturbed problems is, in general, inapplicable for practical calculations. As shown in [4], in the case of a singularly perturbed ordinary differential convection-diffusion equation, standard schemes on uniform grids are not ε-uniformly well-conditioned and, as a consequence, ε-uniformly stable to perturbations in the data of the grid problem, in particular, to computer perturbations.

In the present talk, in the case of a singularly perturbed parabolic convection-diffusion equation, we consider an approach to study stability of solutions of the standard difference scheme on the uniform grid in the presence of perturbations in the data of the grid problem and/or computer perturbations. We have received the conditions imposed on the perturbations which depend on ε, N, N_0, under which the perturbed grid solution converges as N, $N_0 \to \infty$; here N and N_0 are the numbers of intervals in uniform meshes in x and t, respectively, [5]. These conditions allow you to use standard finite difference schemes on uniform grids for solving singularly perturbed problems for parabolic convection-diffusion equations in the presence of perturbations.

This research was partially supported by the Russian Foundation for Basic Research under grant No.13-01-00618.

REFERENCES

[1] P. Farrell, A. Hegarty, J. Miller, E. O'Riordan and G. Shishkin. *Robust computational techniques for boundary layers.* CRC Press, Boca Raton, 2000.

[2] G. Shishkin and L. Shishkina. *Difference Methods for Singular Perturbation Problems.* CRC Press, 2009.

[3] A. Samarskii. *Theory of Difference Schemes.* Marcel Dekker, New York, 2001.

[4] G. Shishkin. Stability of a Standard Finite Difference Scheme for a Singularly Perturbed Convection-Diffusion Equation. *Doklady Mathematics*, **87** (1):107–109, 2013.

[5] G. Shishkin. Standard Finite Difference Scheme for a Singularly Perturbed Parabolic Convection-Diffusion Equation under Computer Perturbations. *Doklady Akademii Nauk*, **462** (1): 2015.(in Russian)