

GENERALIZED GREEN'S FUNCTIONS FOR M -TH ORDER DISCRETE NONLOCAL PROBLEMS*

GAILĖ PAUKŠTAITĖ and ARTŪRAS ŠTIKONAS

¹*Institute of Mathematics and Informatics, Vilnius University*

Akademijos str. 4, LT-08663 Vilnius, Lithuania

²*Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko str. 24, LT-03225 Vilnius, Lithuania

E-mail: gaile.paukstaite@mif.stud.vu.lt

We consider m -th order discrete boundary value problems with m nonlocal conditions

$$\begin{aligned}\mathcal{L}u &:= a_i^m u_{i+m} + a_i^{m-1} u_{i+m-1} + \dots + a_i^1 u_{i+1} + a_i^0 u_i = f_i, \quad i = 0, 1, 2, \dots, n - m, \\ \langle L_j, u \rangle &= 0, \quad j = 1, 2, \dots, m,\end{aligned}$$

where nonlocal conditions are described by discrete linear functionals L_j , $j = 1, 2, \dots, m$, $n \geq m$.

For this problem, ordinary Green's function, that describes the unique exact solution, and its properties were investigated in [4]. Here we consider generalized Green's function that describes the minimum norm least squares solution.

Some properties of generalized Green's function for second order discrete problem were investigated in the paper [2]. Now we compare the properties of generalized Green's function that describes the minimum norm least squares solution to the properties of ordinary Green's function that describes the unique exact solution to the m -th order problem [4].

We can prove that generalized Green's function satisfies the same investigated properties of ordinary Green's function but in the sense of least squares.

REFERENCES

- [1] A. Ben-Israel, T.N.E. Greville. *Generalized Inverses. Theory and Applications*. Springer Verlag, New York, 2003.
- [2] G. Paukškaitė, A. Štikonas. Generalized Green's functions for the second order discrete problems with nonlocal conditions. *Lith. Math. J.*, **54** (2), 2014.
- [3] R. Penrose. On best approximate solutions of linear matrix equations. *Proc. Cambridge Philos. Soc.*, **52** 17–19, 1956.
- [4] S. Roman. *Green's functions for boundary-value problems with nonlocal boundary conditions*. Vilnius University, 2011. Doctoral dissertation

*The research was partially supported by the Research Council of Lithuania (grant No. MIP-047/2014).