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ON THE DISCRETE UNIVERSALITY OF THE PERIODIC HURWITZ ZETA-FUNCTION

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Let $s = \sigma + it$ be a complex variable, $\alpha, 0 < \alpha \leq 1$, be a fixed parameter, and let $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers with minimal period $q \in \mathbb{N}$. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and can be continued meromorphically to the whole complex plane.

The universality of the function $\zeta(s, \alpha; \mathfrak{a})$ with transcendental α was obtained in [1]. The discrete universality of $\zeta(s, \alpha; \mathfrak{a})$ also with transcendental α was considered in [2]. In this report, we propose a theorem on discrete universality of $\zeta(s, \alpha; \mathfrak{a})$ under new hypotheses.

THEOREM 1. *Suppose that the set*

$$\left\{ \Big(\log(m+\alpha) : m \in \mathbb{N}_0 \Big), \frac{\pi}{h} \right\}$$

is linearly independent over the field of rational numbers, K is a compact subset of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complement, and $f(s)$ is a continuous non-vanishing function on K which is analytic in the interior of K. Then, for every $\epsilon > 0$,

$$\liminf_{N \to \infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s \in K} \Big| \zeta(s + ikh, \alpha; \mathfrak{a}) - f(s) \Big| < \epsilon \right\} > 0.$$

REFERENCES

[1] A. Javtokas, A. Laurinčikas. Universality of the periodic Hurwitz zeta-function. *Integral Transforms Spec. Funct.*, **17** (10):711–712, 2006.

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