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ANALYTICAL SOLUTION TO A PROBLEM ON MHD FLOW IN A RECTANGULAR DUCT UNDER SLIP CONDITION ON ITS SIDE WALLS

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We consider the magnetohydrodynamic (MHD) problem on a fully developed flow of a conducting fluid in a duct with the rectangular cross-section, located in a uniform external magnetic field, and under a slip condition on both side walls of the duct. The flow is driven by a constant pressure gradient.

The fully developed flows in rectangular ducts are well studied for different electric conductivities of the walls ([1]), but under "no slip" condition on the duct walls. In [2] three classic MHD problems are revisited on assuming a hydrodynamic slip condition at the interface between the electrically conducting fluid and the insulating walls. One of the problems studied analytically in [2] is the problem on a fully developed flow in the rectangular duct with insulating walls and slip conditions on the Hartmann walls (the walls perpendicular to the magnetic field).

We present the analytical solution to the problem on a fully developed flow of a conducting fluid in the rectangular duct with the perfectly conducting Hartmann walls and non-conducting side walls (the walls parallel to the external magnetic field) with the slip boundary condition on the side walls. The slip condition is given by the 3rd kind boundary condition ([2]).

The dimensionless MHD equations, describing the problem, have the form ([1], [2]):

$$\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial y^2} + 1 + Ha\frac{\partial b_x}{\partial z} = 0, \qquad \frac{\partial^2 b_x}{\partial z^2} + \frac{\partial^2 b_x}{\partial y^2} + Ha\frac{\partial U}{\partial z} = 0 \tag{1}$$

where $\overrightarrow{V} = U(y,z)\overrightarrow{e_x}$ is the velocity of the fluid; $\overrightarrow{b} = b_x(y,z)\overrightarrow{e_x}$ is the induced magnetic field; $Ha = B_0 h\sqrt{\sigma/\rho\nu}$ is the Hartmann number, which characterizes the ratio of electromagnetic force to viscous force; σ , ρ , ν are the conductivity, the density and the viscosity of the fluid, respectively.

The boundary conditions are

$$z = \pm 1: \; U = 0, \; \frac{\partial b_x}{\partial z} = 0 \quad and \quad y = \pm d: \; U \pm \alpha\frac{\partial U}{\partial y} = 0, \; b_x = 0. \tag{2}$$

The problem is solved analytically by using the Fourier cosine and sine series expansion of the velocity and the induced magnetic field.

REFERENCES

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