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ON INTEGRATION MATRIX DEFINED BY CHEBYSHEV DIFFERENTIATION MATRIX

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The concept of the differentiation matrix was derived from pseudospectral methods and nowadays it has proven to be a very useful tool in numerical solution of differential equations [1].

By differentiation matrix D_N the numerical differentiation process of function $f(x)$ is performed as a matrix-vector product $\vec{f'} = D_N \vec{f}$, where $\vec{f}$ is the vector of function values and $\vec{f'}$ is the vector of approximate derivative values at the given points $x_k, k = 0, ..., N$. The Chebyshev differentiation matrix is obtained using an interpolating polynomial on Chebyshev-Gauss-Lobatto points $x_k = -\cos\frac{\pi k}{N}$, $k = 0, ...N$ (or Lobatto grid), which results in minimum error and avoids Runge's phenomenon associated with uniform grid.

Matrix D_N is always degenerate and therefore has no inverse.

In [2] for differentiation matrix D_N was defined, so called, pseudoinverse matrix B_N by the equality

$$B_N D_N = E_N - I_N \tag{1}$$

where E_N is the identity matrix and all elements of I_N are zeroes except first column of ones.

Using (1) for solving initial problems of ordinary differential equations we, in fact, convert numerically this problem to solving equivalent integral equation, where matrix B_N is discrete approximation of corresponding indefinite integral in this equation. Such integration matrix technique can be applied also for solving different problems with partial differential equations and this usually leads to more accurate numerical results [3] than applying D_N directly.

In present work the explicit formulas for elements of matrix B_N defined by Chebyshev differentiation matrix are obtained and the following properties previously observed only numerically are proven:

1.The elements in the last row of the matrix B_N coincide with weights in Clenshaw-Curtis quadrature.

2.The max-norm of the matrix B_N is equal to 2 for any number N and hence is not growing with N contrary case of equispaced grid.

3.The sum of all eigenvalues of matrix B_N also does not depend on N and is equal to 1 for any N.

REFERENCES

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