JOINT DISCRETE UNIVERSALITY OF HURWITZ ZETA-FUNCTIONS

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The Hurwitz zeta-function $\zeta(s,\alpha)$, $s = \sigma + it$, with parameter $\alpha$, $0 < \alpha \leq 1$, is defined, for $\sigma > 1$, by the series

$$\zeta(s,\alpha) = \sum_{m=0}^{\infty} \frac{1}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. It is known that Hurwitz zeta-functions $\zeta(s,\alpha_1),\ldots,\zeta(s,\alpha_r)$ are jointly universal in the sense that their shifts $\zeta(s+i\tau,\alpha_1),\ldots,\zeta(s+i\tau,\alpha_r)$, $\tau \in \mathbb{R}$, approximate simultaneously a given collection of analytic functions. More precisely, let $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$, $\mathcal{K}$ be the class of compact subsets of $D$ with connected complements, and let $H(K)$, $K \in \mathcal{K}$, be the class of continuous functions which are analytic in the interior of $K$. Then the following result is known [1].

**Theorem 1.** Suppose that the set $\{\log(m + \alpha_j) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, j = 1,\ldots,r\}$ is linearly independent over the field of rational numbers $\mathbb{Q}$. For $j = 1,\ldots,r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H(K_j)$. Then, for every $\varepsilon > 0$,

$$\liminf_{T \to \infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0,T] : \sup_{1 \leq j \leq r} \sup_{s \in K_j} |\zeta(s+i\tau,\alpha_j) - f_j(s)| < \varepsilon \right\} > 0.$$

In the report, we consider the so-called joint discrete universality of Hurwitz zeta-functions. In this case, a collection of analytic functions is simultaneously approximated by shifts $\zeta(s + ikh,\alpha_1),\ldots,\zeta(s + ikh,\alpha_r)$ with fixed $h > 0$ and $k \in \mathbb{N}_0$. The following theorem is true [1].

**Theorem 2.** Suppose that the set $\{(\log(m + \alpha_j) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, j = 1,\ldots,r) : \pi\}$ is linearly independent over $\mathbb{Q}$. For $j = 1,\ldots,r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H(K_j)$. Then, for every $\varepsilon > 0$,

$$\liminf_{N \to \infty} \frac{1}{N+1} \# \left\{ 0 \leq k \leq N : \sup_{1 \leq j \leq r} \sup_{s \in K_j} |\zeta(s + ikh,\alpha_j) - f_j(s)| < \varepsilon \right\} > 0.$$

Also, the approximation by shifts $\zeta(s + ikh_1,\alpha_1),\ldots,\zeta(s + ikh_r,\alpha_r)$ will be discussed.

**REFERENCES**