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JOINT DISCRETE UNIVERSALITY OF HURWITZ ZETA-FUNCTIONS

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The Hurwitz zeta-function $\zeta(s,\alpha)$, $s=\sigma+it$, with parameter α, $0<\alpha\leq 1$, is defined, for $\sigma>1$, by the series

$$\zeta(s,\alpha)=\sum_{m=0}^{\infty}\frac{1}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. It is known that Hurwitz zeta-functions $\zeta(s,\alpha_1),\dots,\zeta(s,\alpha_r)$ are jointly universal in the sense that their shifts $\zeta(s+i\tau,\alpha_1),\dots,\zeta(s+i\tau,\alpha_r)$, $\tau\in\mathbb{R}$, approximate simultaneously a given collection of analytic functions. More precisely, let $D=\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\}$, $\mathcal{K}$ be the class of compact subsets of D with connected complements, and let $H(K)$, $K\in\mathcal{K}$, be the class of continuous functions which are analytic in the interior of K. Then the following result is known [1].

THEOREM 1. *Suppose that the set* $\{\log(m+\alpha_j):\ \ m\in\mathbb{N}_0=\mathbb{N}\cup\{0\},\ j=1,\dots,r\}$ *is linearly independent over the field of rational numbers* $\mathbb{Q}$. *For* $j=1,\dots,r$, *let* $K_j\in\mathcal{K}$ *and* $f_j(s)\in H(K_j)$. *Then, for every* $\varepsilon>0$,

$$\liminf_{T\to\infty}\frac{1}{T}\mathrm{meas}\left\{\tau\in[0,T]:\ \sup_{1\leq j\leq r}\sup_{s\in K_j}|\zeta(s+i\tau,\alpha_j)-f_j(s)|<\varepsilon\right\}>0.$$

In the report, we consider the so-called joint discrete universality of Hurwitz zeta-functions. In this case, a collection of analytic functions is simultaneously approximated by shifts $\zeta(s+ikh,\alpha_1),\dots,\zeta(s+ikh,\alpha_r)$ with fixed $h>0$ and $k\in\mathbb{N}_0$. The following theorem is true [1].

THEOREM 2. *Suppose that the set* $\left\{(\log(m+\alpha_j):\ m\in\mathbb{N}_0=\mathbb{N}\cup\{0\},\ j=1,\dots,r),\frac{\pi}{h}\right\}$ *is linearly independent over* $\mathbb{Q}$. *For* $j=1,\dots,r$, *let* $K_j\in\mathcal{K}$ *and* $f_j(s)\in H(K_j)$. *Then, for every* $\varepsilon>0$,

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\leq k\leq N:\ \sup_{1\leq j\leq r}\sup_{s\in K_j}|\zeta(s+ikh,\alpha_j)-f_j(s)|<\varepsilon\right\}>0.$$

Also, the approximation by shifts $\zeta(s+ikh_1,\alpha_1),\dots,\zeta(s+ikh_r,\alpha_r)$ will be discussed.

REFERENCES

[1] A. Laurinčikas. Joint discrete universality of Hurwitz zeta-functions. *Sbornik: Mathematics*, **205** (11):1599–1619, 2014.