ON HIGHER DEGREE F-TRANSFORMS BASED ON B-SPLINES

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The concept of F-transforms (direct and inverse) was introduced in [1] and further extended to the higher degree F-transform [2] (so called $F^m$-transforms). The $F^m$-transforms have been successfully applied in image processing, data analysis and time series analysis during last years. We focus on approximation properties of the $F^m$-transform with respect to a generalized uniform fuzzy partition given by B-splines of odd degree. Let an interval $[a, b]$ be fixed and $k, N \in \mathbb{N}$. By $A$ we denote the central B-spline of order $2k-1$ with integer knots $-k, \ldots, k$. Let $h = (b-a)/(N+2k)$, $t_i = a + h(i+k)$, $i = -k, \ldots, N+k$; then $a = t_{-k} < t_0 < t_N < t_{N+k} = b$. Functions $A_0, \ldots, A_N$, where $A_i(t) = A((t-t_i)/h)$, $i = 0, \ldots, N$, form a $(h, hk)$-uniform generalized fuzzy partition of interval $[a, b]$ in the sense of [3]. Our construction generalizes the commonly used uniform fuzzy partition given by a generating function of the triangular shape [2].

Our main result is as follows. Suppose that $p$ is a polynomial, $\deg p \leq 2k-1$ and $m$ is a non-negative integer s.t. $2m+1 \geq \deg p$. Then $p$ coincides with its inverse $F^m$-transform (w.r.t. the $(h, hk)$-uniform generalized fuzzy partition based on B-splines of degree $2k-1$) $F^m(p, \cdot)$ on interval $[t_{k-1}, t_{N-k+1}]$.

Based on this result we obtain error estimations and prove that using B-splines improves the quality of approximation of a function by its inverse $F^m$-transform. If integers $r, m$ satisfy $0 \leq m \leq k-1$, $1 \leq r \leq 2m+2$, then for $f \in C^r[a, b]$ it holds that

$$|f(x) - F^m(f, x)| \leq O(h^r) \quad \text{for all } x \in [t_{k-1}, t_{N-k+1}].$$

We consider the inverse $F^m$-transform in the context of local approximation methods given by splines and compare the obtained result with approximation properties of other local methods.

REFERENCES


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