Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

ON HIGHER DEGREE F-TRANSFORMS BASED ON B-SPLINES[1]

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The concept of F-transforms (direct and inverse) was introduced in [1] and further extended to the higher degree F-transform [2] (so called F^m-transforms). The F^m-transforms have been successfully applied in image processing, data analysis and time series analysis during last years.

We focus on approximation properties of the F^m-transform with respect to a generalized uniform fuzzy partition given by B-splines of odd degree. Let an interval $[a, b]$ be fixed and $k, N \in \mathbb{N}$. By A we denote the central B-spline of order $2k-1$ with integer knots $-k, \dots, k$. Let $h = (b-a)/(N+2k)$, $t_i = a + h\,(i+k)$, $i = -k, \dots, N+k$; then $a = t_{-k} < t_0 < t_N < t_{N+k} = b$. Functions $A_0, \dots, A_N$, where $A_i(t) = A\left((t-t_i)/h\right)$, $i = 0, \dots, N$, form a (h, hk)-uniform generalized fuzzy partition of interval $[a, b]$ in the sense of [3]. Our construction generalizes the commonly used uniform fuzzy partition given by a generating function of the triangular shape [2].

Our main result is as follows. Suppose that p is a polynomial, $\deg p \leq 2k-1$ and m is a non-negative integer s.t. $2m+1 \geq \deg p$. Then p coincides with its inverse F^m-transform (w.r.t. the (h, hk)-uniform generalized fuzzy partition based on B-splines of degree $2k-1$) $\mathcal{F}^m(p, \cdot)$ on interval $[t_{k-1},\ t_{N-k+1}]$.

Based on this result we obtain error estimations and prove that using B-splines improves the quality of approximation of a function by its inverse F^m-transform. If integers r, m satisfy $0 \leq m \leq k-1$, $1 \leq r \leq 2m+2$, then for $f \in C^r[a, b]$ it holds that

$$|f(x) - \mathcal{F}^m(f, x)| \leq O(h^r) \quad \text{for all } x \in [t_{k-1},\ t_{N-k+1}].$$

We consider the inverse F^m-transform in the context of local approximation methods given by splines and compare the obtained result with approximation properties of other local methods.

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[1]This work has been supported by the European Social Fund within the project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045.