Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

EXPONENTIAL TYPE SPLINES FOR SOLUTIONS OF DIFFUSION PROBLEM IN MULTILAYERED DOMAIN

HARIJS KALIS, ILMARS KANGRO AND ULDIS STRAUTINS

Institute of Mathematics and Computer Science of University of Latvia

Raiņa bulvāris 29, Rīga LV-1459, Latvija

E-mail: `kalis@lanet.lv, kangro@ru.lv, strautins@lu.lv`

We propose an averaging method for solving a 3-D boundary-value problem in multilayer domain. We introduce an exponential spline, which respects the mean integral values of a piece-wise smooth function. Consider the nonstationary 3-D problem of the linear diffusion theory for multilayered piece-wise homogenous materials of N layers in the subdomains
$\Omega_i = \{(x,y,z) : x \in (0,L_x), y \in (0,L_y), z \in (z_{i-1}, z_i)\}, i = \overline{1,N}$,
where $H_i = z_i - z_{i-1}$ is the height of the layer $\Omega_i, z_0 = 0, z_N = L_z$.

We find the functions $c_i = c_i(x,y,z,t)$ in every layer Ω_i for $(x,y,z) \in \Omega_i$ at time t by solving a 3-D initial-boundary value problem with piece-wise diffusion coefficients D_{iz} . We use the averaging method with respect to z using special exponential or hyperbolic trigonometric functions
$c_i(x,y,z,t) = c_{iz}(x,y,t) + m_{iz}(x,y,t)\frac{0.5H_i \sinh(a_i(z-\overline{z_i}))}{\sinh(0.5a_iH_i)} + e_{iz}G_{iz}\left(0.25\frac{\sinh^2(a_i(z-\overline{z_i}))}{\sinh^2(0.5a_iH_i)} - A_{i0z}\right)$,
where
$c_{iz}(x,y,t) = \frac{1}{H_i}\int_{z_{i-1}}^{z_i} c_i(x,y,z,t)dz,\ G_{iz} = \frac{H_i}{D_{iz}},\quad A_{i0z} = 0.25\frac{\sinh(a_iH_i)/(a_iH_i)-1}{\cosh(a_iH_i)-1} \in [0, 1/12]$,
$\overline{z_i} = (z_{i-1}+z_i)/2,\ z \in [z_{i-1}, z_i],\ i = \overline{1,N}$.
In the limit as the parameters $a_i > 0$ tend to zero, one obtains the integral parabolic spline proposed by A. Buikis [1] (see also [2]):
$c_i(x,y,z,t) = c_{iz}(x,y,t) + m_{iz}(x,y,t)(z-\overline{z_i}) + e_{iz}(x,y,t)G_{iz}\left(\frac{(z-\overline{z_i})^2}{H_i^2} - \frac{1}{12}\right)$.
The unknown functions $m_{iz}(x,y,t)$, $e_{iz}(x,y,t)$ are determined by the boundary conditions in z-direction, but the averaged values $c_{iz}(x,y,t)$ can be obtained from the reduced 2D problem. The parameters a_i can be chosen to minimize the error of the solution.

This approach reduces problems of mathematical physics in 3 spatial dimensions with discontinuous coefficients to problems in 2 spatial dimensions. The same procedure can be applied in another direction to obtain a 1D problem, which can be solved analytically.

The solutions of the obtained 2-D initial-boundary value problem in a single layer is also obtained numerically, using the implicit finite difference approximation and alterating direction (ADI) method of Douglas and Rachford. The numerical solution is compared with the analytical solution . [1].

REFERENCES

[1] A. Buikis. The approximation with splines for problems in layered systems. *Riga, Acta Universitatis Latviensis,*, **592** 135-138, 1994.in Latvian

[2] I.Kangro, H.Kalis, A.Gedroics, E. Teirumnieka and E. Teirumnieks. On mathematical modelling of metals distribution in peat layer. *Mathematical Modelling and Analysis*, **19** 4:568-588, 2014.

[1]This work was partially supported by the grant 623/2014 of the Latvian Council of Science