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JOINT MIXED LIMIT THEOREM FOR A CLASS OF ZETA-FUNCTIONS

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For $m \in \mathbb{N}$, attach $g(m) \in \mathbb{N}$, and, for $j \in \mathbb{N}$ with $1 \le j \le g(m)$, let $f(j,m) \in \mathbb{N}$ and $a_m^{(j)} \in \mathbb{C}$. Denote by p_m the mth prime number, and let $s = \sigma + it$ be a complex variable. The zeta-function $\varphi(s)$ introduced by the second author [2] is defined by the polynomial Euler product

$$\varphi(s) = \prod_{m=1}^{\infty} A_m^{-1}(p_m^{-s}), \tag{1}$$

where A_m's are polynomials given by $A_m(x) = \prod_{j=1}^{g(m)} (1 - a_m^{(j)} x^{f(j,m)})$. Suppose that $g(m) \le c p_m^{\alpha}$, $|a_m^{(j)}| \le p_m^{\beta}$ with $c > 0$, and some non-negative constants α and β. The infinite product (1) converges absolutely for $\sigma > \alpha + \beta + 1$.

Let $\mathfrak{B} = \{b_m : m \in \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers with minimal period $l \in \mathbb{N}$, and let $\gamma \in \mathbb{R}$, $0 < \gamma \le 1$, be a fixed parameter. Then the function $\zeta(s, \gamma; \mathfrak{B})$ introduced by A. Laurinčikas and A. Javtokas [1] is defined, for $\sigma > 1$, by the series

$$\zeta(s, \gamma; \mathfrak{B}) = \sum_{m=0}^{\infty} \frac{b_m}{(m+\gamma)^s}.$$

From the periodicity of $\mathfrak{B}$ we have $\zeta(s, \gamma; \mathfrak{B}) = \frac{1}{l^s} \sum_{k=0}^{l-1} b_k \zeta(s, (k+\gamma)/l)$, $\sigma > 1$, where $\zeta(s, \gamma)$ is the classical Hurwitz zeta-function. Therefore, the function $\zeta(s, \gamma; \mathfrak{B})$ is a linear combination of the functions $\zeta(s, \gamma)$, and last equality gives analytic continuation for $\zeta(s, \gamma; \mathfrak{B})$ to the whole complex plane, where it is regular, except, maybe, for a simple pole at $s = 1$ with residue $b := \frac{1}{l} \sum_{k=0}^{l-1} b_k$.

In the talk, we will discuss on joint mixed limit theorem in the space of holomorphic functions for the collection of functions $\varphi(s)$ and $\zeta(s, \gamma; \mathfrak{B})$, when special additional conditions are fulfilled.

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