EXISTANCE AND UNIQUENESS OF THE QUADRATIC/LINEAR RATIONAL SPLINE HISTOPOLANT

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Let the points x_i be such that $a = x_0 < x_1 < \ldots < x_n = b$ and let z_i , $i = 1, \ldots, n$, be given real numbers. We choose spline knots ξ_i , $i = 1, \ldots, n$, such that $\xi_1 = x_0, x_1 < \xi_2 < x_2, \ldots, x_{i-1} < \xi_i < x_i, \ldots, x_{n-2} < \xi_{n-1} < x_{n-1}, \xi_n = x_n$, and want to construct a C^2 function S on [a, b] such that it is quadratic/linear rational function on every subinterval $[\xi_i, \xi_{i+1}]$ satisfying the histopolation conditions

$$\int_{x_{i-1}}^{x_i} S(x) dx = z_i (x_i - x_{i-1}), \quad i = 1, \dots, n.$$
(1)

In addition to (1), we impose two boundary conditions.

We show that the quadratic/linear rational spline histopolant is unique. Linear/linear rational histopolating spline of class C^1 and combined comonotone histopolating spline of class C^1 always exists (see [1] and [2]). In case of quadratic/linear rational spline histopolation we can prove the existence of histopolant only in the case if the moments $M_i = S''(\xi_i), i = 1, ..., n$, are satisfying certain conditions.

We also show that quadratic/linear rational histopolating spline may not exist for certain strictly convex data even on uniform mesh.

Some samples will be shown.

REFERENCES

- M.Fischer, P.Oja. Monotonicity preserving rational spline histopolation. J. Comput. Appl. Math., 175 195–208, 2005.
- [2] M.Fischer, P.Oja and H.Trossmann. Comonotone shape-preserving spline histopolation. J. Comput. Appl. Math., 200 (1):127–139, 2007.