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# ON SOME FUNCTIONS DEFINED BY ORDINARY DIFFERENTIAL EQUATIONS

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We consider functions $S(t)$ and $C(t)$ which solve Cauchy problems

$$x'' = (\alpha^2 - 1)x - 2\alpha^2 x^3, \quad x(0) = 0, \quad x'(0) = 1$$

and

$$x'' = (\alpha^2 - 1)x - 2\alpha^2 x^3, \quad x(0) = 1, \quad x'(0) = 0$$

respectively, where $0 \leq \alpha \leq 1$.

If $\alpha = 0$, then $S(t)$ coincides with trigonometric sine $\sin t$, but $C(t)$ coincides with trigonometric cosine $\sin t$. If $\alpha = 1$, then $S(t)$ coincides with lemniscatic sine $\mathrm{sl}\, t$, but $C(t)$ coincides with lemniscatic cosine $\mathrm{cl}\, t$.

We use Jacobi elliptic function technique and provide different formulae for the functions $S(t)$ and $C(t)$ such as nonlinear unit circle formula:

$$\forall t \in \mathbb{R}: \quad S^2(t) + \alpha^2 S^2(t)C^2(t) + C^2(t) = 1,$$

addition and double argument formulae and others.

**REFERENCES**

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