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ZETA FUNCTIONS. UNIVERSALITY, ZEROS, AND MOMENTS

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Let $s = \sigma + it$ be a complex variable. The Riemann zeta-function is defined by the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \qquad (\sigma > 1).$$

B. Riemann famously related the distribution of prime numbers to the distribution of zeros of the Riemann zeta-function $\zeta(s)$. In 1975, S. M. Voronin proved his well known Universality theorem of the Riemann zeta-function, which states that any non-vanishing analytic function can be approximated uniformly by certain purely imaginary shifts of the Riemann zeta-function in the strip $1/2 < \sigma < 1$. The Riemann zeta-function is the most famous among many other important zeta-functions considered in the analytic number theory.

In this talk, we present results related to universality theorem, to the zero distribution and moments of various zeta-functions. In particular, we are interested in the effective version of the universality theorem, also in the possibility to extend the class of universal zeta-functions. An interesting example is the universality of the Selberg zeta-function. The universality is closely connected to the zero distribution of zeta-functions. Universality of the Lerch zeta-function shows that this function usually has infinitely many zeros in the strip $1/2 < \sigma < 1$. We also investigate classifications of zeros of $\zeta(s)$. Some results on zeros were suggested by computer investigations of the related zeta-function. Sometimes explicit computer calculations were used in the proofs. For these and other allied results the authors received the Lithuanian National Science Award (2015).