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ON THE EIGENVALUE PROBLEM WITH BOUNDARY CONDITION CONTAINING A PARAMETER

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We consider the eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega, \quad \frac{\partial u}{\partial \nu} + \alpha u = 0 \quad \text{on} \quad \Gamma, \tag{1}$$

where $\Omega \subset R^n$, $n \geq 1$, is a bounded domain with boundary Γ. By ν we denote the outward unit normal vector to Γ, α is a real parameter. There is a sequence of eigenvalues $\lambda_1(\alpha) < \lambda_2(\alpha) \leq \ldots$ of the problem (1) enumerated according to their multiplicities with $\lim\limits_{k\to\infty} \lambda_k(\alpha) = +\infty$. Also, we consider the sequence of eigenvalues $0 < \lambda_1^D < \lambda_2^D \leq \ldots$ of the Dirichlet eigenvalue problem

$$\Delta u + \lambda u = 0 \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on} \quad \Gamma \tag{2}$$

with $\lim\limits_{k\to\infty} \lambda_k^D = +\infty$. We study the behavior of $\lambda_k(\alpha)$ for large positive values of α.

THEOREM 1. *Let $\Gamma \in C^2$. Then the eigenvalues $\lambda_k(\alpha)$ satisfy the estimates*

$$\lambda_k^D - C_1 \frac{\left(\lambda_k^D\right)^2}{\alpha} \leq \lambda_k(\alpha) \leq \lambda_k^D, \quad \alpha > \alpha_1 > 0, \quad k = 1, 2, \ldots, \tag{3}$$

where the constants C_1 and α_1 does not depend on k.

THEOREM 2. *Let $\Gamma \in C^3$. Then*

$$\lambda_1(\alpha) = \lambda_1^D - \frac{\int_\Gamma \left(\frac{\partial u_1^D}{\partial \nu}\right)^2 ds}{\int_\Omega \left(u_1^D\right)^2 dx} \frac{1}{\alpha} + o\left(\frac{1}{\alpha}\right), \quad \alpha \to +\infty, \tag{4}$$

where u_1^D is the first eigenfunction of the Dirichlet problem (2).

The relation (4) shows that first power of α in the denominator in (3) can not be replaced by $\alpha^{1+\delta}$ with any $\delta > 0$.

REFERENCES

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