Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

AN ALTERNATIVE APPROACH TO LOWER AND UPPER APPROXIMATION OPERATORS

ALEKSANDRS EĻKINS[1] AND ALEKSANDRS ŠOSTAK[2]

[1] *Departament of Mathematics, University Of Latvia*

Zeļļu iela 8, Rīga LV-1002, Latvija

[2] *Institute of Mathematics and CS*

Raiņa bulv. 29, Rīga LV-1459, Latvija

E-mail: `aleksandrs.elkinsi@gmail.com, aleksandrs.sostaks@lumii.lv`

Let R be a reflexive transitive relation on a set X, and A be an L-subset of X, that is a mapping $A : X \to L$ where $L = (L, \leq, \wedge, \vee, *, \mapsto)$ is a cl-monoid [1]. Following the works of many authors (see e.g. [2], [3], [4]) by a lower and upper approximations of an L-set A we call respectively

$$l(A)(x) = \inf_{x' \in X} (R(x,x') \mapsto_A (x')) \; \forall A \in L^X, \; u(A)(x) = \sup_{x' \in X} (R(x,x') * A(x')) \; \forall A \in L^X.$$

An important property of this approximation is that $l(A)(x) \leq A(x) \leq u(A)(x) \; \forall A \in L^X$. In case L is an MV-algebra [5], residuation $\mapsto$ satisfies a double negation property $(a \mapsto 0) \mapsto 0 = a \; \forall a \in L$ and hence $a \mapsto 0$ could be viewed as a complement of element a [5]. This allows to get a "good" relation between lower and upper approximation operators: $l(A)(x) \mapsto 0 = u(A \mapsto 0)(x)$. This property has a clear counterpart in case of topology, where l and u approximation operators could be realized as interior and closure operators respectively. This inspires us to define now operators l_u and u_l starting with l and u and setting

$$l_u(A)(x) = (u(A \mapsto 0)(x)) \mapsto 0, \; u_l(A)(x) = (l(A \mapsto 0)(x)) \mapsto 0,$$

which we view as alternative approaches of lower and upper approximations. In case of MV-algebra they obviously coincide with l and u respectively.
We study properties of operators l_u and u_l, and their relations with "classical" operators l and u. In particularly we show that the inequalities $l \leq l_u$ and $u \leq u_l$ are always true.

Acknowlegement The second named author kindly announces the support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045

REFERENCES

[1] G. Birkhoff, Lattice Theory, AMS Providence, RI, 1995

[2] D. Dubois, H. Prade. Rough fuzzy sets and fuzzy rough sets, *Internat. J. General Systems*, **17** (2–3) 191-209, 1990.

[3] A.M. Radzikowska, E.E. Kerre. A comparative study of fuzzy rough sets, *Fuzzy Sets and Systems.* **126**, 137-155, 2002.

[4] A.Eļkin, A. Šostak, On some categories of approximate systems generated by L-relations. Proceedings of Workshop on Rough Sets Theory, 41-49, 2011.

[5] C.C. Chang, Algebraic analysis of many-valued logics. Transactions of the American Mathematical Society 88: 476490, 1958