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ON OPTIMALITY OF ADAPTIVE TIME INTEGRATION METHODS FOR SOLVING SYSTEMS OF ODE

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We consider the initial value problem

$$\frac{dU}{dt} = F(U), \quad 0 < t \leq T, \quad U(0) = U_0.$$

The method of lines can be used to approximate PDEs and to get a system of ODEs. Here we will restrict to two simple but reliable and efficient time integration algorithms to solve ODEs: the backward Euler method (BEM) and the Discontinuous Galerkin Method (DGM).

Let us define the space of piecewise constant functions on $I = (0, 1]$:

$$S_h(I) := \big\{\varphi : I \to R^d, \ \ \varphi|_{I_n} \in P_0(I_n)\big\}, \quad I_n = (t_{n-1}, t_n].$$

The dG(0) approximation $U \in S_h(I)$ is determined by requiring that

$$\sum_{n=1}^{N} \Big\{ \int_{I_n} \big(U^{'} - F(U), \psi\big) dt + \big(U_{n-1}^{+} - U_{n-1}^{-}, \psi_{n-1}^{+}\big) \Big\} = 0$$

for all $\psi \in S_h(I)$.

The review on adaptive time integration strategies and derivation of guaranteed error estimates of the discrete solutions are presented in [1; 2], see also references given these works.

Our aim is to investigate the optimality of the obtained adaptive discrete time meshes. This analysis is based on dual problem techniques (or the adjoint method) and a posteriori error estimates. The comparison of different adaptive methods includes also some well-known adaptive strategies based on local error control methods.

Results of computational experiments are presented.

REFERENCES

[1] W. Bangerth, R. Rannacher. *Adaptive Finite Element Methods for Differential Equations.* Lectures in Mathematics, ETH Zurich, Birkhauser, 2003.

[2] Y. Gao and L. Petzold. A posteriori error estimation and global error control for ordinary differential equations by the adjoint method. *SIAM J. Sci. Comput.*, **26** (2):359–374, 2004.