

DECOMPOSITION METHOD FOR A BLOCK-TRIDIAGONAL MATRIX SYSTEM

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Study of the three-body scattering includes developing the reliable analytical approaches as well as effective parallel computational techniques. One of the rigorous approaches for treating the three-body scattering problems above the breakup threshold is based on the configuration space Faddeev formalism [1]. It reduces the scattering problem to the boundary value problem by implementing appropriate boundary conditions. These boundary conditions have been introduced by S. P. Merkuriev [2] and their new representation has recently been constructed in Ref. [3]. The numerical solution of the boundary value problem [2; 3] for the two-dimensional integro-differential Faddeev equations in polar coordinates includes solving the linear system with a block-tridiagonal matrix of large order. The block-tridiagonal structure arises from an expansion of the unknown solution in a basis of the Hermite splines on the θ -grid and the finite-difference approximation of the second derivative on the equidistant ρ -grid. The matrix sweeping algorithm [4] is traditionally used for such problems. The algorithm is well defined and robust for matrices with diagonal dominance, but its sequential nature makes it difficult to be applied for parallel computational systems.

The present talk is focused on the developed decomposition method (DM) for efficient parallel solving the block-tridiagonal matrix system. The initial matrix is logically reduced to some new independent on-diagonal blocks and a coupling matrix of much smaller size [5]. The solution includes parallel inversions of the on-diagonal blocks and solving the equation for the coupling matrix. The DM includes using the sweeping algorithm for dealing with the blocks, but the parallel structure and possible recursivity of the method lead to remarkable growth of the performance. The analytical estimation of the computational speedup of the DM with respect to the sequential sweeping algorithm and practical tests have been carried out. For not so large supercomputing systems, the DM allows us to obtain a linear growth of the computational speedup S with increase of the number of computing units P : $S \simeq 3/7 P$. For larger supercomputing systems, the nonparallelized part of the method is increased, so the linear growth decelerates. The application of the DM for parallelization of solving the linear system of equations reduces the overall time of calculation up to 10 times.

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