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CLASSIFICATION OF SOLUTIONS TO THE 4–ORDER SINGULAR EMDEN–FOWLER TYPE EQUATIONS

IRINA ASTASHOVA

Lomonosov Moscow State University

Leninskie Gory, GSP-1, Moscow, 1199911, Russia

E-mail: ast@diffiety.ac.ru

The asymptotic classification is given of all possible solutions to the equation

$$y^{\mathrm{IV}}(x) - p_0\,|y|^k \operatorname{sgn} y = 0, \quad 0 < k < 1,\ p_0 > 0. \tag{1}$$

Cf. [1](7.1, 7.3) and [2] for $k > 1$. A solution $u : (a,\, b) \to \mathbb{R}$ with $-\infty \le a < b \le +\infty$ is called a *MUE-solution* if the following conditions hold: (i) the equation has no solution equal to u on some subinterval of $(a,\, b)$ and not equal to u at some point of $(a,\, b)$; (ii) either there is no solution defined on another interval containing $(a,\, b)$ and equal to u on $(a,\, b)$ or there exist at least two such solutions not equal to each other at points arbitrary close to the boundary of $(a,\, b)$.

Theorem 1. *Suppose $0 < k < 1$ and $p_0 > 0$. Then all MUE-solutions to equation (1) are divided into the following thirteen types according to their asymptotic behavior.*

1–2. Defined on $(b, +\infty)$ (up to the sign) solutions with the power asymptotic behavior near the boundaries of the domain (with the relative signs $\pm$): $y(x) \sim \pm C_{4k}\,(x-b)^{-\frac{4}{k-1}}, \quad x \to b+0$, $y(x) \sim \pm C_{4k}\, x^{-\frac{4}{k-1}}, \quad x \to +\infty$, where $C_{4k} = \left(\frac{4(k+3)(2k+2)(3k+1)}{p_0\,(k-1)^4}\right)^{\frac{1}{k-1}}$.

3–4. Defined on semi-axes $(-\infty, b)$ (up to the sign) solutions with the power asymptotic behavior near the boundaries of the domain (with the relative signs $\pm$): $y(x) \sim \pm C_{4k}\,|x|^{-\frac{4}{k-1}}, \quad x \to -\infty$, $y(x) \sim \pm C_{4k}\,(b-x)^{-\frac{4}{k-1}}, \quad x \to b-0$.

5. Defined on the whole axis periodic oscillatory solutions. All of them can be received from one, say $z(x)$, by the relation $y(x) = \lambda^4 z(\lambda^{k-1}x + x_0)$ with arbitrary $\lambda > 0$ and x_0. So, there exists such a solution with any maximum $h > 0$ and with any period $T > 0$, but not with any pair (h, T).

6–7. Defined on $(-\infty, +\infty)$ solutions which are oscillatory as $x \to -\infty$ and have the power asymptotic behavior near $+\infty$: $y(x) \sim \pm C_{4k}(p(b))\,(b-x)^{-\frac{4}{k-1}}, \quad x \to b-0$. For each solution a finite limit of the absolute values of its local extrema exists as $x \to -\infty$.

8–9. Defined on $(-\infty, +\infty)$ solutions which are oscillatory as $x \to +\infty$ and have the power asymptotic behavior near near $-\infty$: $y(x) \sim \pm C_{4k}(p(b))\,(x-b)^{-\frac{4}{k-1}}, \quad x \to b+0$. For each solution a finite limit of the absolute values of its local extrema exists as $x \to +\infty$.

10–13. Defined on $(-\infty, +\infty)$ solutions which have the power asymptotic behavior near $-\infty$ and $+\infty$: $y(x) \sim \pm C_{4k}(p(b))\,|x|^{-\frac{4}{k-1}}, \quad x \to \pm\infty$.

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