Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

NUMERICAL MODELING OF ANOMALOUS DIFFUSION PROCESSEN IN MULTIDIMENSIONAL DOMAIN

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In present work we discuss the mathematical models of anomalous diffusion within the framework of fractional differential equations [1; 2; 3]:

$$\frac{\partial^\beta u}{\partial t^\beta} = \sum_{r=1}^{p} \left[q_r \frac{\partial^{\alpha_r}}{\partial_+ x_r^{\alpha_r}} + (1-q_r) \frac{\partial^{\alpha_r}}{\partial_- x_r^{\alpha_r}} \right] u + f(x,t), \quad x = (x_1, \ldots, x_p) \in G,\ t > 0,$$

where $G = \{x = (x_1, \ldots, x_p) \,|\, 0 < x_r < l_r,\ r = \overline{1,p}\}$, $1 < \alpha_r < 2$, $0 < \beta < 1$, $r = \overline{1,p}$.

We compare different methods [2]-[5] of numerical solution of model problems with boundary conditions of the first kind and exemplify main ideas of proposed methods. Various definitions of fractional derivatives and their approximation to different orders of accuracy are also considered. All schemes are proved to be unconditionally stable. Comparison of results allows us to conclude that obtained solutions with different values of α_r and β poses all properties of anomalous diffusion.

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