Abstract

In this work we present a theory for the description of the non-equilibrium loading of a quantum dot with one energy level. Using non-equilibrium Green function approach we calculate the time-dependence of the average number of electrons on the dot as well as the fluctuations of this number. We are able to isolate the essential parameters that affect the charge transfer in the quantum pump. The presented model can be used to analyze experimentally obtained volt-ampere characteristics of real quantum pumps and to enhance the precision of currently available devices.

Quantum pumping

Quantum pumps are electrostatically controlled nano-sized current sources that produce an integer number of electrons per cycle and therefore can be used, for example, as standard currents. A common setup for a quantum pump consists of a quantum dot, attached to leads and separated from them by potential barriers that are controlled by time-dependent voltages applied to gate electrodes. The loading and unloading of the quantum dot are essentially time-dependent and often non-adiabatic processes; therefore an appropriate mathematical description of charge transfer through the quantum pump is a challenging task [1, 2].

Model and formalism

Quantum dot, attached to leads, can be described with Hamiltonian \( H = H_{\text{dot}} + H_{\text{leads}} + H_L \), where \( H_{\text{dot}} = \sum_{\alpha} \varepsilon_{\alpha} c_{\alpha}^\dagger c_{\alpha} \), \( H_{\text{leads}} = \sum c_{\alpha}^\dagger c_{\alpha} \), \( H_L = \sum \sqrt{\frac{2}{m^*}} \phi_{\alpha} d_{\alpha}^\dagger d_{\alpha} + h.c. \). The charge capture process in the dot can be treated using non-equilibrium Green function approach. For case when both the energy of quantum dot \( \varepsilon \) and tunneling amplitudes \( \Gamma \) are time-dependent, the average occupation of the dot is

\[
\langle n(t) \rangle = \langle \frac{1}{1 + e^{(\varepsilon - \mu + \Gamma(t))/kT}} \rangle
\]

Here \( G(\varepsilon,t') = \frac{1}{2\pi} \int d\omega G(\varepsilon^{\dagger}(\omega),\omega,t') \) is Green function, \( f(\mu) \) is Fermi function for electrons in the leads with chemical potential \( \mu \) and temperature \( T \).

To characterize the fluctuations of the number of electrons on the dot one can use its variation \( \sigma_n^2 = n(1-n) \).

Loading & backtunneling

To model charge pumping process, we introduce following time dependencies for parameters \( \varepsilon \) and \( \Gamma \):

\[
\varepsilon(t) = \varepsilon_c + \xi(t-t_c)
\]

\[
\Gamma(t) = \frac{h}{2\pi} \delta(t-t_c) / \tau
\]

Here \( \varepsilon_c \) is proportional to the applied voltage \( V_2 \) and \( \xi \) characterizes the time-dependency of the voltage \( V_1 \). Time \( t_c \) can be considered as the charge-capture moment. The backtunneling of the electrons occurs at \( t_c + \tau - (\mu - \varepsilon) / \xi \).

Essential parameters

In our model the volt-ampere characteristic of the quantum pump is determined by only three essential parameters: temperature in the leads \( k_B T \), inverse pinch-off time \( h / \tau \), plunger-to-barrier ratio \( \xi / \tau \), each of them can be associated with some uncertainty of the electron energy. In the 3D plots below the dependencies of the occupation fluctuations on applied control voltage and the dominating parameter \( (k_B T, h / \tau , \xi / \tau) \) are shown.

THERMAL UNCERTAINTY is the uncertainty of the energy of electrons in the leads. With increase of temperature, energy distribution of electrons is smeared and consequently the fluctuations are observed in wider range of applied voltages.

If \( k_B T < h / \tau \) and \( k_B T > \xi / \tau \), the final average occupation of the dot can be calculated as

\[
\langle n(t) \rangle = \frac{1}{1 + e^{(\varepsilon - \mu + \Gamma(t))/kT}}
\]

CLASSICAL UNCERTAINTY is the uncertainty of the energy of the quantum dot due to applied time-dependent voltage. Fluctuations of the occupation increases, if the energy of the dot is changed faster.

Plunger-to-barrier ratio \( \xi / \tau \) is a measure for the speed of changes of the energy on dot that does not depend on pumping frequency. If \( \xi / \tau > k_B T \) and \( \xi / \tau > h / \tau \), the final average occupation of the dot can be calculated as

\[
\langle n(t) \rangle = e^ {- (\varepsilon - \mu) / (k_B T)}
\]

QUANTUM UNCERTAINTY is the uncertainty of the energy of the quantum dot due to finite duration of the contact between dot and left lead (i.e., due to the fact that the barrier between dot and lead is closing). The speed of barrier closing can be characterized using inverse pinch-off time \( h / \tau \).

If \( h / \tau > k_B T \) and \( h / \tau > \xi / \tau \), the final average occupation of the dot is

\[
\langle n(t) \rangle = \frac{2}{\pi} \arctan e^{ - (\varepsilon - \mu) / (h / \tau)}
\]

Plunger-to-barrier ratio \( \xi / \tau \) is device-specific parameter that does not depend on pumping frequency. The inverse pinch-off time \( h / \tau \), in turn, is proportional to the frequency, therefore, by scanning the pumping frequency, the relative importance of classical and quantum uncertainties can be easily changed.

In the 3D plot on the left side the dependency of the occupation fluctuations on applied control voltage and the ratio \( (h / \tau) / (\xi / \tau) \) is shown for \( k_B T = 0 \).