Symmetric XOR Games

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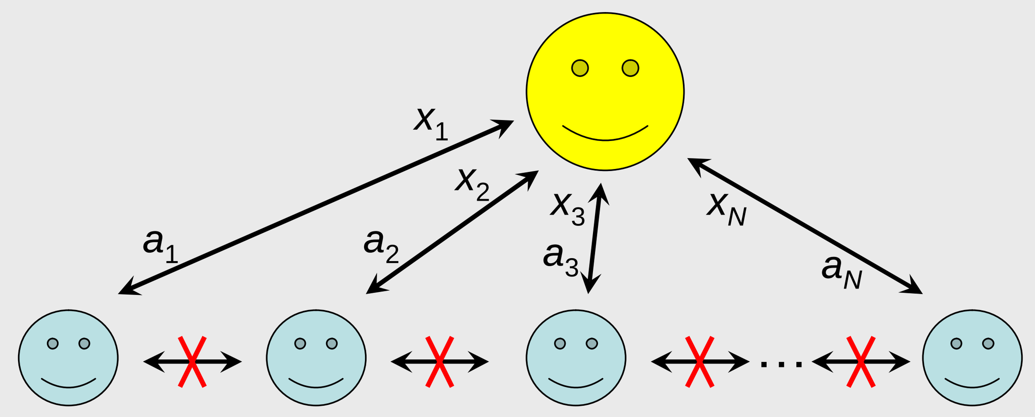
Definitions

- Game between *referee* & number of *players*
- Players receive questions $a_1, a_2, \ldots, a_N$
- Players are prohibited to communicate
- Players must return answers $x_1, x_2, \ldots, x_N$



If all a_i and x_i are 1-bit values, then:

- outcome of **symmetric game** depends only on $\sum a_i$ and $\sum x_i$
- outcome of **XOR game** depends only on $a_1, a_2, \ldots, a_N$ and parity of $\sum x_i$

Complexities

Classical version

Each player must choose his answers for each of 2 inputs. Totally, he has 4 choices: **00**, **01**, **10**, **11**.

- Winning probability of *arbitrary XOR* game is NP-hard
- Winning probability of *symmetric* game seems to be $O(N^7)$
- Winning probability of *symmetric XOR* game is $O(N^3)$

Quantum version

All players share quantum system in state $|0\ldots0\rangle + |1\ldots1\rangle$.
Each player chooses local operations for each of 2 inputs.
Each player answers according to his measurement result.

- Optimal strategy for *arbitrary XOR* game is somewhere in *N*-dimensional space [Werner, Wolf, 2001]
- Optimal strategy for *symmetric XOR* game is somewhere In 1-dimensional space [Ambainis et al., 2010]

Search for optimal strategy

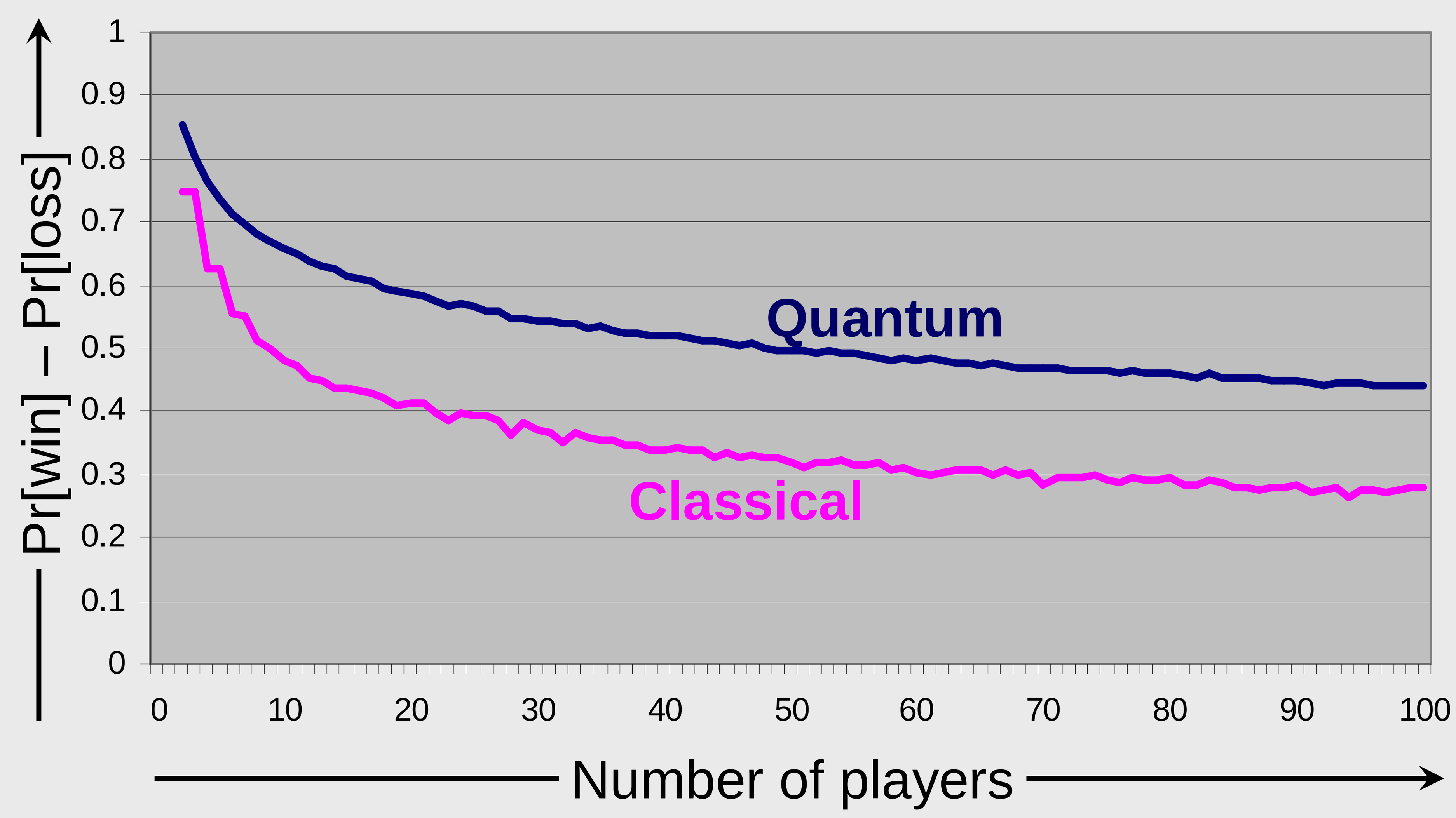
Werner and Wolf showed that outcome for arbitrary XOR game is

$$\max_{l_1,l_2,\mathbf{K},l_N}\left|\frac{1}{2^N}\sum_{a_1,\mathbf{K},a_N\in\{0,1\}}\Gamma(a_1,\mathbf{K}\,a_N)\,l_1^{a_1}l_2^{a_2}\,\mathbf{K}\,l_N^{a_N}\right|$$

Ambainis et al. showed that outcome for symmetric XOR game is

$$\max_{l}\left|\frac{1}{2^N}\sum_{k=0}^{N}\Gamma(k)\binom{N}{k}l^k\right|$$

Mean of outcomes for symmetric XOR games



Arbitrary symmetric input distribution in CHSH

CHSH is typical symmetric XOR game. We study its modification, where referee produce inputs with any probabilities $[P_{00},P_{01},P_{10},P_{11}]$ but with restriction of symmetry: $P_{01} = P_{10}$.

Optimal classical strategy is one of the following:

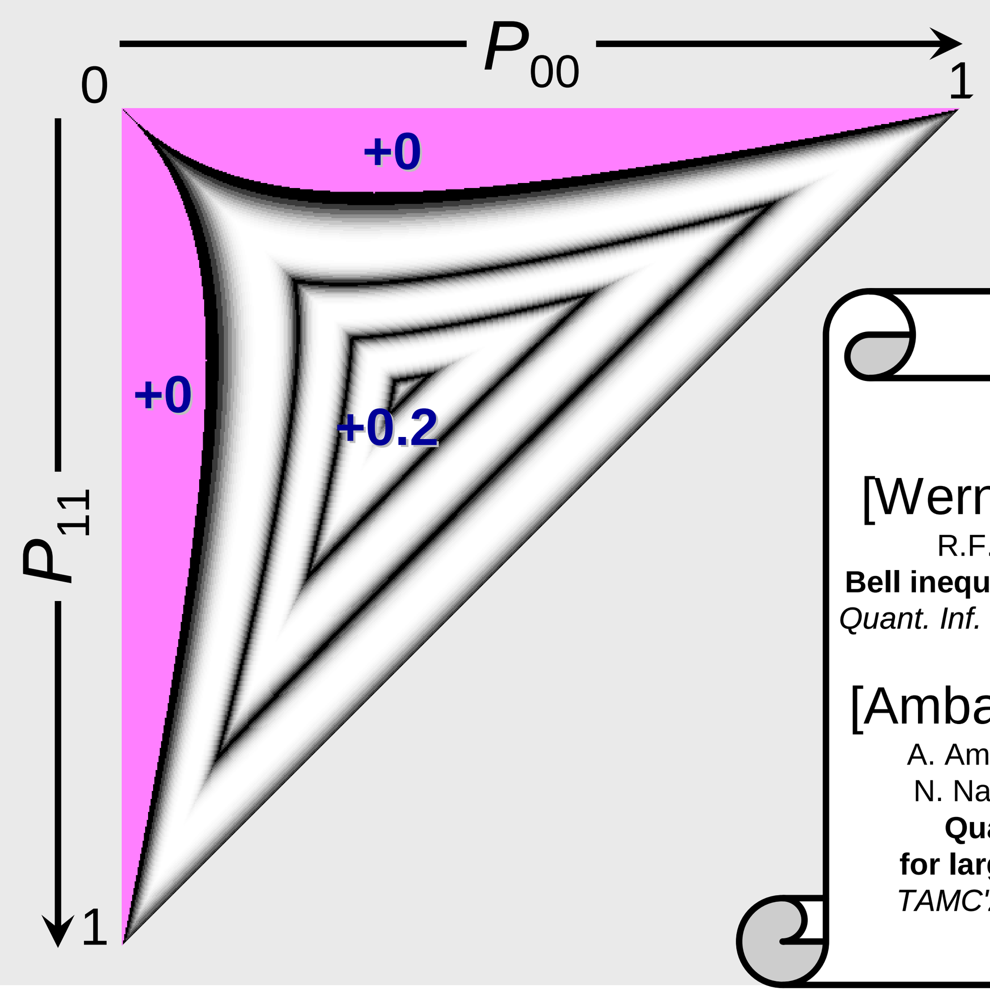
Answers of player 1		Answers of player 2		Inputs, for which strategy gives correct answer
Input 0	Input 1	Input 0	Input 1	
0	0	0	0	0,0 0,1 1,0 ~~1,1~~
0	1	0	0	0,0 0,1 ~~1,0~~ 1,1
0	1	0	1	0,0 ~~0,1~~ 1,0 1,1
0	1	1	0	~~0,0~~ 0,1 1,0 1,1

It gives linear outcome: $1-2\min(P_{00},P_{01},P_{10},P_{11})$

Optimal quantum outcome can be expressed as:

$$\frac{P_{00}+P_{11}}{2}\sqrt{\frac{(P_{00}+P_{11})^2+(1-2P_{00})(1-2P_{11})}{P_{00}P_{11}}}$$

Quantum violations for all P_{00}, P_{11}



References

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