IEGULDĪJUMS TAVĀ NĀKOTNĒ

Language Varieties and Quantum Automata

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Outline

- Motivation & approach
- Monoids, varieties an regular languages
  - Relation to automata and “forbidden” constructions
- Existing results on automata
  - Automata with classical acceptance (using algebraic methods)
  - Automata with decide and halt acceptance (forbidden constructions)
- New results – R and R1 and MM-QFA

Quantum Finite Automata

- One-way quantum finite automaton (QFA) $A = (Q, \Sigma, q_0, \delta)$ is specified by a finite set of states Q, a finite input alphabet Σ, an initial state $q_0 \in Q$, and a transition function

$$\delta : Q \times \Gamma \times Q \rightarrow C_{[0,1]},$$

  where $\Gamma = \Sigma \cup \{\#, \$\}$ is the input tape alphabet of A and #, $ are end-markers not in Σ. For any input letter σ, the transition function is determined by a $|Q| \times |Q|$ unitary matrix V_σ , where $(V_\sigma)_{i,j} = \delta(q_j, \sigma, q_i)$.
- *Compare:*
  - **Deterministic automata**: V_σ has exactly one 1 in each row and the rest are 0
  - **Probabilistic automata**: V_σ is a stochastic matrix
- Acceptance type
  - Classic - *Classical acceptance".* Consider an automaton with the set of states partitioned into final states and non-final states. It is said that an automaton accepts (rejects) an input "classically", if (1) the computation is halted as soon as the last letter of an input word has been read; (2) the input is accepted, if the automaton has entered a final state when halted, and rejected otherwise.
  - *"Decide-and-halt acceptance".* Consider an automaton with the set of states partitioned into non-halting states and halting states, where halting states are further classified as accepting states or rejecting states. It is said that an automaton accepts (rejects) an input in a "decide-and-halt" manner, if (1) the computation is halted as soon as the automaton enters a halting state; (2) the input is accepted, if the automaton has entered an accepting state; (3) the input is rejected, if the automaton has entered a rejecting state.
- Computation
  - Apply transformation according to the read letter to the state of automata
  - Optional: Perform specific measurement on the resulting state
  - Continue with next letter

Automata Models

	"Classical" word acceptance	"Decide-and-halt" word acceptance
Quantum Finite Automata with mixed states	Latvian Quantum Finite Automata (LQFA) [Ambainis et al, Golovkins and Kravtsev]	Enhanced Quantum Finite Automata (EQFA) [Nayak]
Probabilistic Reversible Automata	"Classical" Probabilistic Reversible Automata (C-PRA) [Golovkins and Kravtsev]	"Decide-and-halt" Probabilistic Reversible Automata (DH-PRA) [Golovkins and Kravtsev]
Quantum Finite Automata with pure states	Measure-Once Quantum Finite Automata (MO-QFA) [Moore et al]	Measure-Many Quantum Finite Automata (MM-QFA) [Kondacs and Watrous]
Deterministic Reversible Automata	Group Automata (GA)	Reversible Finite Automata (RFA) [Ambainis and Freivalds]

$$\text{EQFA} \stackrel{?}{=} \text{DH-PRA} \stackrel{?}{=} \text{MM-QFA} \supset \text{RFA}$$

$$\text{LQFA} = \text{C-PRA} \supset \text{MO-QFA} = \text{GA}$$

Motivation

- Automata with decide and halt acceptance
  - Different properties then for classical acceptance: non closure under union and intersection. however closed under inverse homomorphism and word quotient
  - Similar properties for PRA-DH and MM-QFA
- Forbidden constructions
  - An easy way to define languages which can not be recognized by some automata
  - Still the **gaps** for decide and halt automata: if there is no forbidden construction we don't know if the language can be recognised by QFA or not
- Algebraic theory
  - Proofs on automata with classical acceptance
- Approach
  - Separate relevant classes of languages into some subclasses
  - Identify characteristics of subclasses
  - Study these subclasses and prove their properties starting from simpler and then trying to generalize

Algebraic theory

Finite Monoids

- Monoid – a set equipped with an associative operation and an identity element.
- A monoid element s such that $s^2 = s$ is called an *idempotent.*
- Given $s \in M$, exists k such that s^k is an idempotent.

Therefore every element of a finite monoid generates an idempotent. An idempotent generated by s is denoted as s^ω.

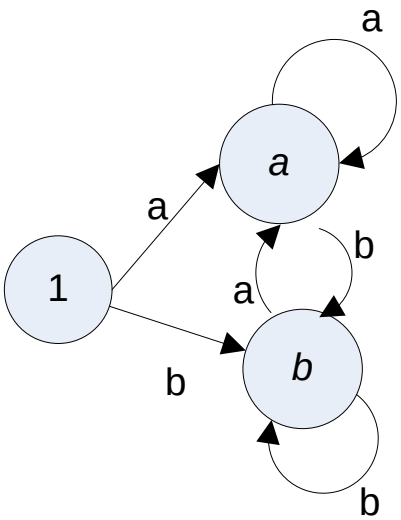
Finite Automata and Finite Monoids

- Recognition of regular languages by finite automata:
  - *Well known*
- Recognition of regular languages by finite monoids:
  - a monoid M recognises a language $L \in A^*$ iff exists a morphism φ from A^* to M such that $L\varphi\varphi^{-1} = L$

Example:
Monoid

	1	*a*	*b*
1	1	*a*	*b*
a	*a*	*a*	*b*
b	*b*	*a*	*b*

Language for the M
a) Let us take e.g. A={a,b} and
a→a ; *b*→ b
b) construct automata
c) arbitrary accepting states



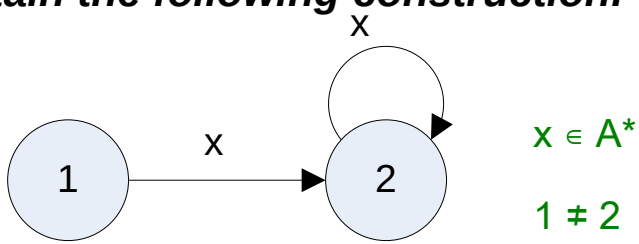
Language Varieties and Monoid Varieties

- Language variety is a class of languages that is:
  - closed under complement, intersection and union;
  - closed under inverse morphisms; (morphism φ: $\forall a,b \in A\ (ab)\varphi = (a\varphi)(b\varphi)$)
  - closed under quotient operations; (quotent: $a^{-1}L = \{v \in A^* \mid av \in L\}$)
- Monoid variety is a set of monoids that is:
  - closed under product;
  - closed under division.
- By Eilenberg theorem, there is a remarkable one-to-one relationship between the set of regular language varieties and the set of finite monoid varieties:
  - monoids in a monoid variety recognise exactly the languages of the corresponding language variety.

Varieties and forbidden constructions

Language L is in this variety iff the complete minimal deterministic automaton of L does not contain the following construction:

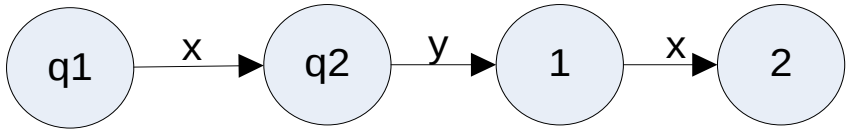
- **G** = $\{x^\omega = 1\}$, the variety of groups;



$x \in A^*$

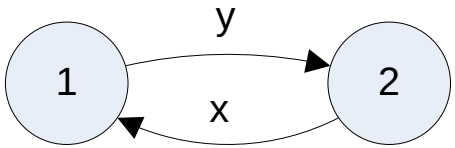
$1 \neq 2$

- $\mathbf{R_1} = \{xyx = xy\}$, the variety of idempotent and *R*-trivial monoids;



$x,y \in A^*$
$1 \neq 2$

- **R** = $\{(xy)^\omega x = (xy)^\omega\}$, the variety of *R*-trivial monoids.



$x,y \in A^*$, $1 \neq 2$

Semi-direct Products

- Let S and T two monoids. A left action of T on S is a map
  $(t, s) \to t \cdot s$ from $T \times S$ into S such that, for all $s, s_1, s_2 \in S$ and $t, t_1, t_2 \in T$,
  1) $(t_1 t_2) \cdot s = t_1(t_2 \cdot s)$
  2) $t \cdot (s_1 + s_2) = t \cdot s_1 + t \cdot s_2$
  3) $1 \cdot s = s$
  4) $t \cdot 0 = 0$

- The semidirect product of S and T (with respect to the given action) is the monoid $S * T$ defined on $S \times T$ by the multiplication
  $(s, t)(s_0, t_0) = (s + t \cdot s_0, t \cdot t_0)$

- A semidirect product $\mathbf{V} * \mathbf{W}$ of two *varieties* $\mathbf{V}$ and $\mathbf{W}$ is the smallest variety containing the monoids $S * T$, such that $S \in \mathbf{V}$ and $T \in \mathbf{W}$.

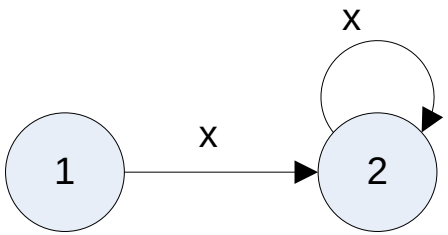
Previous results

Automata with Classical Acceptance 1

The languages recognised by MO-QFA and GA corresponds to the variety

G = $\{x^{\omega} = 1\}$, the variety of groups

A language L is in the language variety above iff the complete minimal deterministic automaton of L does not contain the following constructions:



$x \in A^*$

$1 \neq 2$

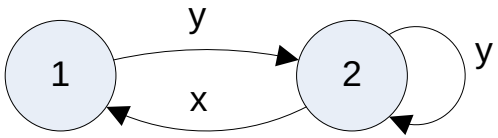
Automata with Classical Acceptance 2

- The languages recognised by LQFA and C-PRA corresponds to the variety

  BG = $\{(x^\omega y^\omega)^\omega = (y^\omega x^\omega)^\omega\}$

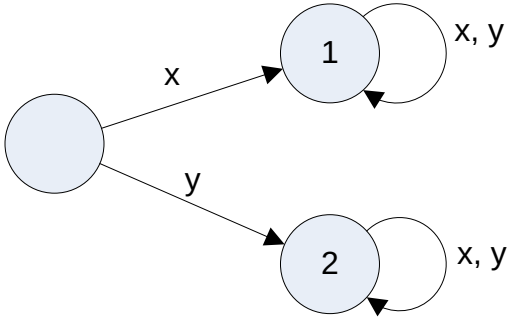
  [Ambainis et al., 2004]

BG = J*G= L*G ^ R*G



$x,y \in A^*$

$1 \neq 2$



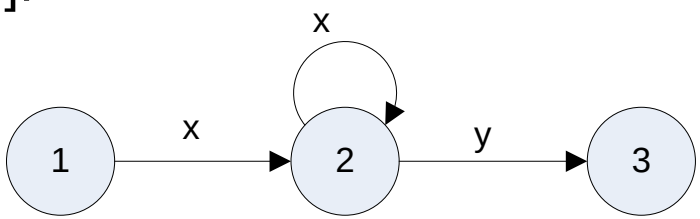
$x,y \in A^*$

$1 \neq 2$

- A language L is in the language variety above iff the complete minimal deterministic automaton of L does not contain the following constructions:

Automata with Decide and Halt Acceptance 1

- A language L is recognised by **RFA** iff the complete minimal deterministic automaton of L does not contain the following construction [Ambainis, Freivalds 98]:

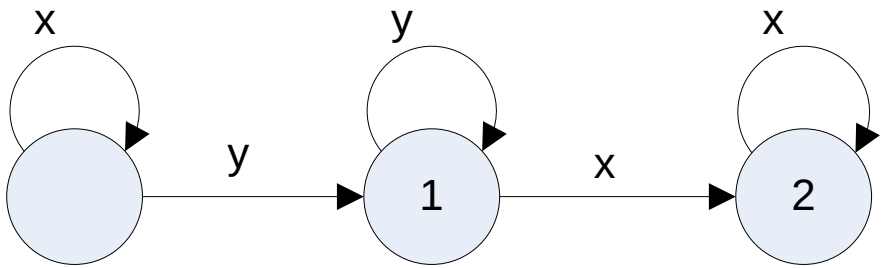


$x,y \in A^*$
$1 \neq 2$
$2 \neq 3$

- Boolean closure of the languages recognised by **RFA** corresponds to the variety

$$\mathbf{R_1 * G} = \mathbf{ER_1} = \{x^\omega y^\omega x^\omega = x^\omega y^\omega\}$$ [Golovkins and Pin, 2006]

- A language L is in the language variety above iff the complete minimal deterministic automaton of L does not contain the following construction:

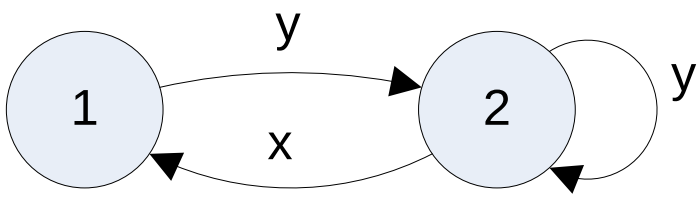


$x,y \in A^*$

$1 \neq 2$

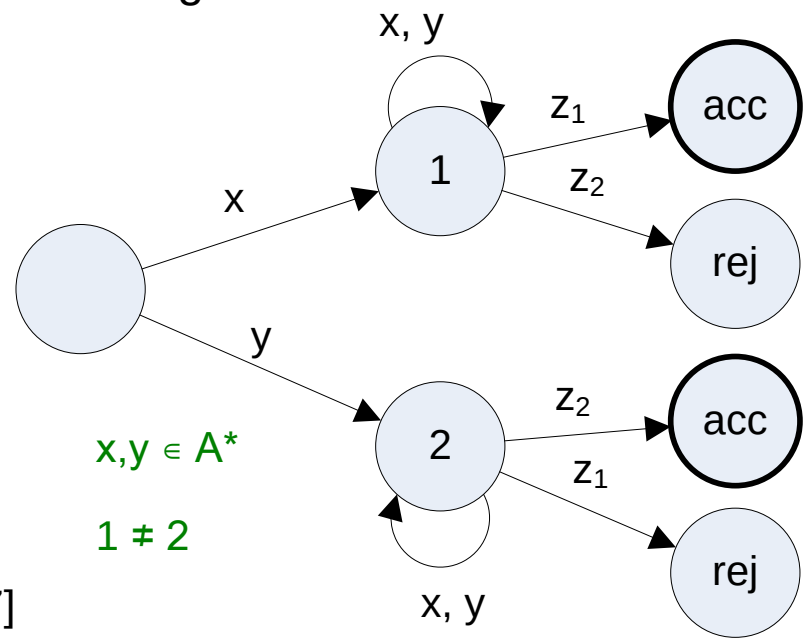
Forbidden Constructions for MM-QFA & DH-PRA

A language L is **NOT** recognised by MM-QFA[DH-PRA] **if** the complete minimal deterministic automaton of L contain one of the following constructions:



$x,y \in A^*$

$1 \neq 2$

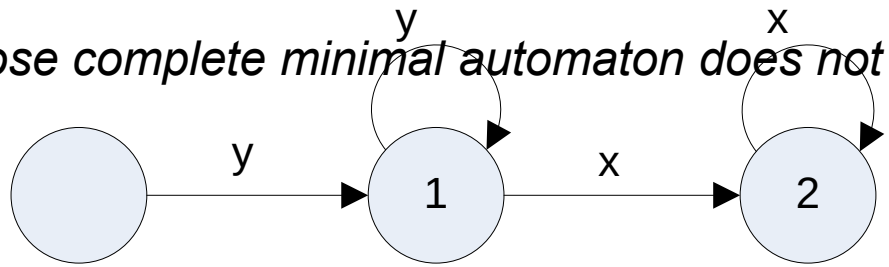


$x,y \in A^*$

$1 \neq 2$

- MM-QFA [Brodsky Pipinger, 99], [Ambainis,Kikusts,Valdats, 01]
- DH-PRA [Golovkins, Kravtsev and Kravcevs, 2007]

[AKV,01] *: A language L whose complete minimal automaton does not contain "two cycles in a row" construction*



$x,y \in A^*$

$1 \neq 2 \neq 3$

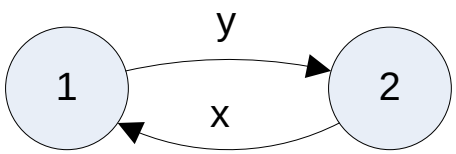
*is recognised by MM-QFA **iff** the complete minimal deterministic automaton of L does not contain one of the forbidden constructions above!*

Present research

R-Trivial Languages 1-2

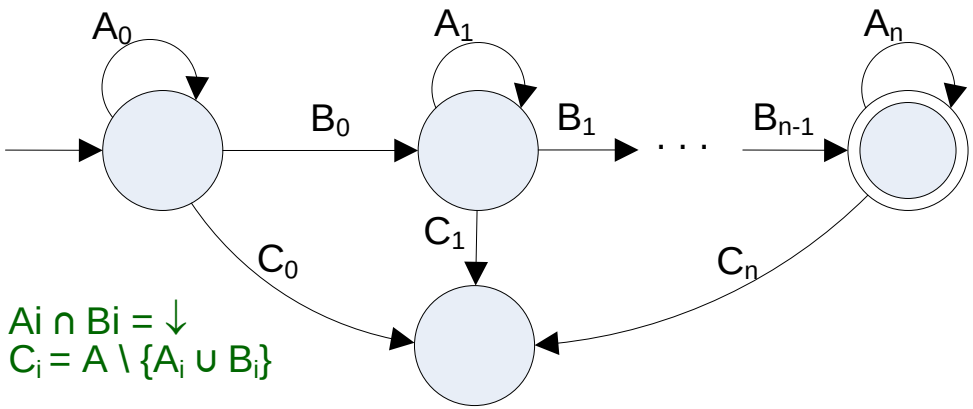
- The languages corresponding to the variety

  R $= \{(xy)^{\omega}x = (xy)^{\omega}\}$
- The languages recognised by a complete minimal deterministic automaton that does not contain the following construction:



$x,y \in A^*,\ 1 \neq 2$

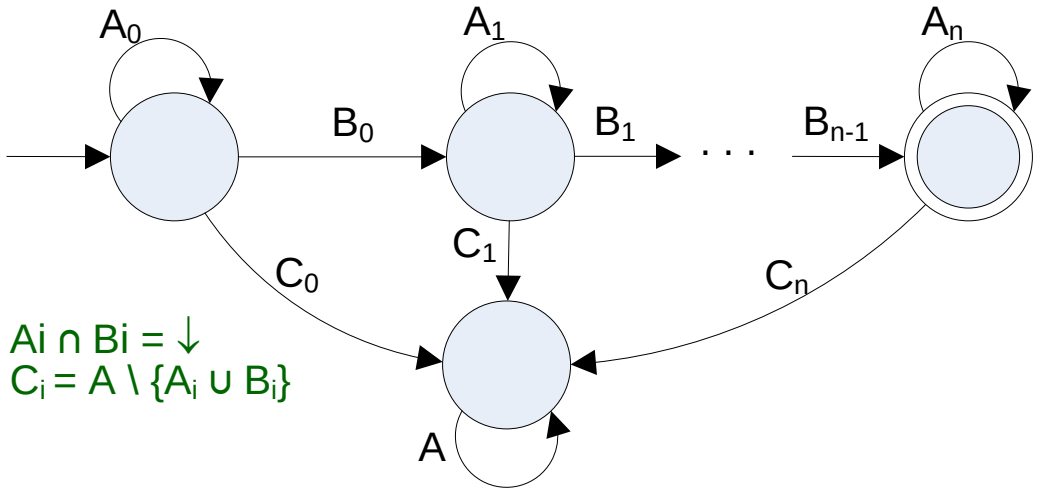
- **Theorem**: Any R-trivial language may be decomposed [Note: possibly in several ways] as a disjoint union of the languages recognised by the following automata :



$Ai \cap Bi = \downarrow$

$C_i = A \setminus \{A_i \cup B_i\}$

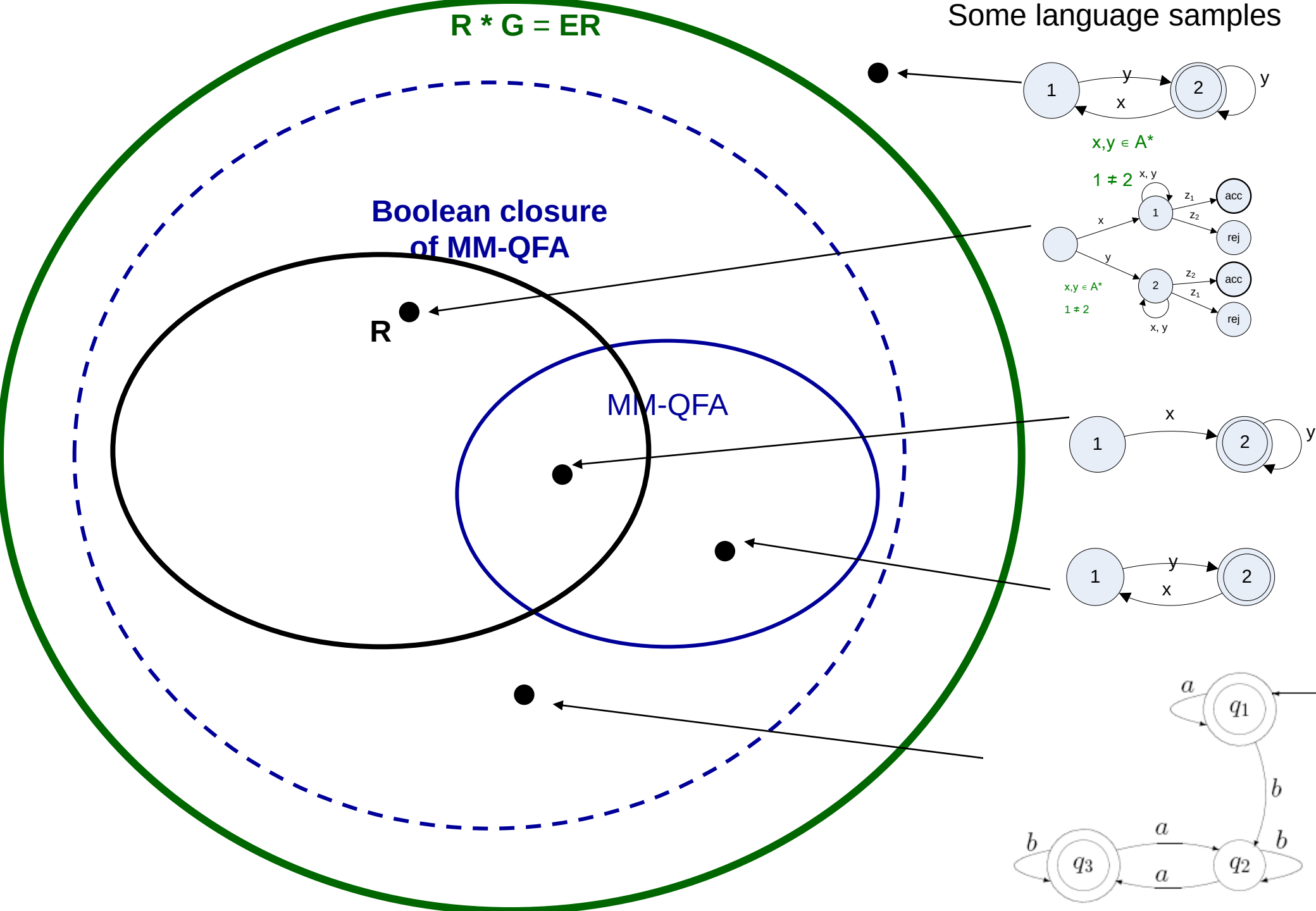
R-Trivial Languages 2-2



- Any language above may be recognised by DH-PRA, MM-QFA and EQFA.
  [Golovkins, Kikusts, Kravcevs, Kravcevs ,2009]

- **Corollary:** Boolean closure of the l\anguages recognised by MM-QFA, DH-PRA and EQFA contains any R-trivial language (i.e., the language in the variety

  R $= \{(xy)^{\omega}x = (xy)^{\omega}\}$

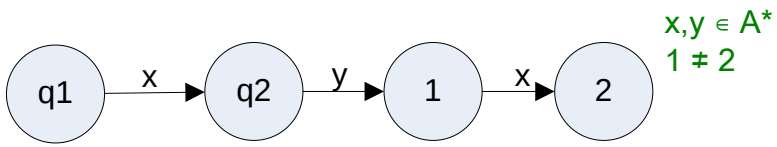

R * G = ER
Some language samples
Boolean closure
of MM-QFA
R
MM-QFA
1
2
y
x
x,y ∈ A*
1 ≠ 2
x, y
z1
z2
acc
rej
a
b
q1
q2
q3

R1 trivial languages 1-2

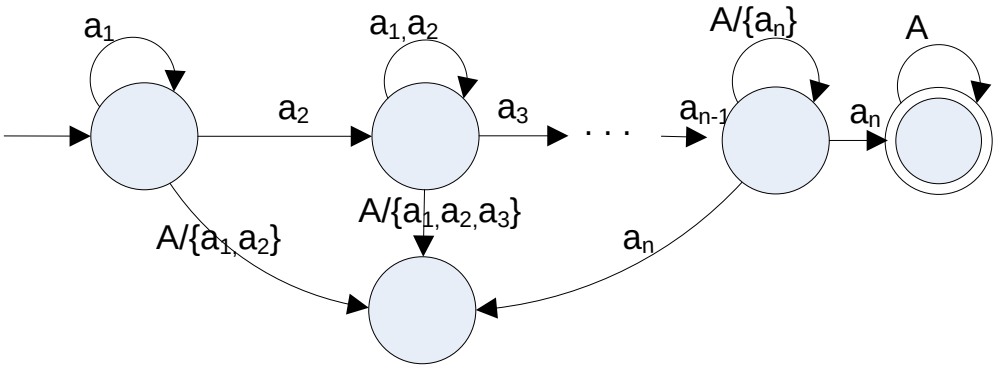
- The languages corresponding to the variety

  $\mathbf{R_1} = \{xyx = xy\}$,

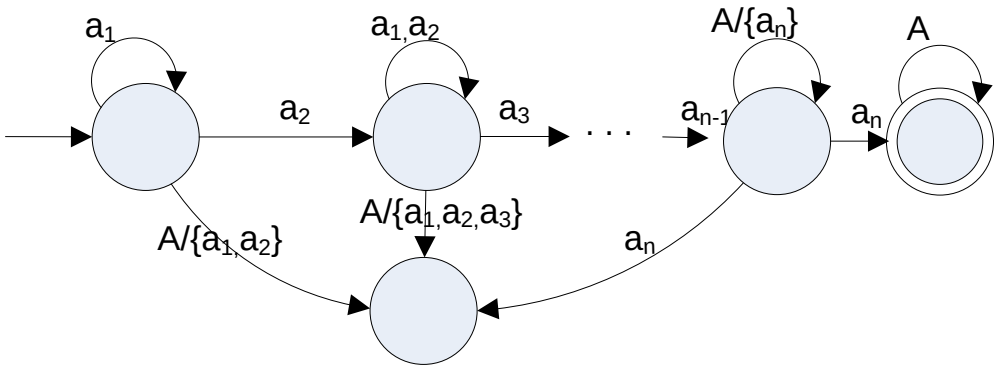
- The languages recognised by a complete minimal deterministic automaton that does not contain the following construction:



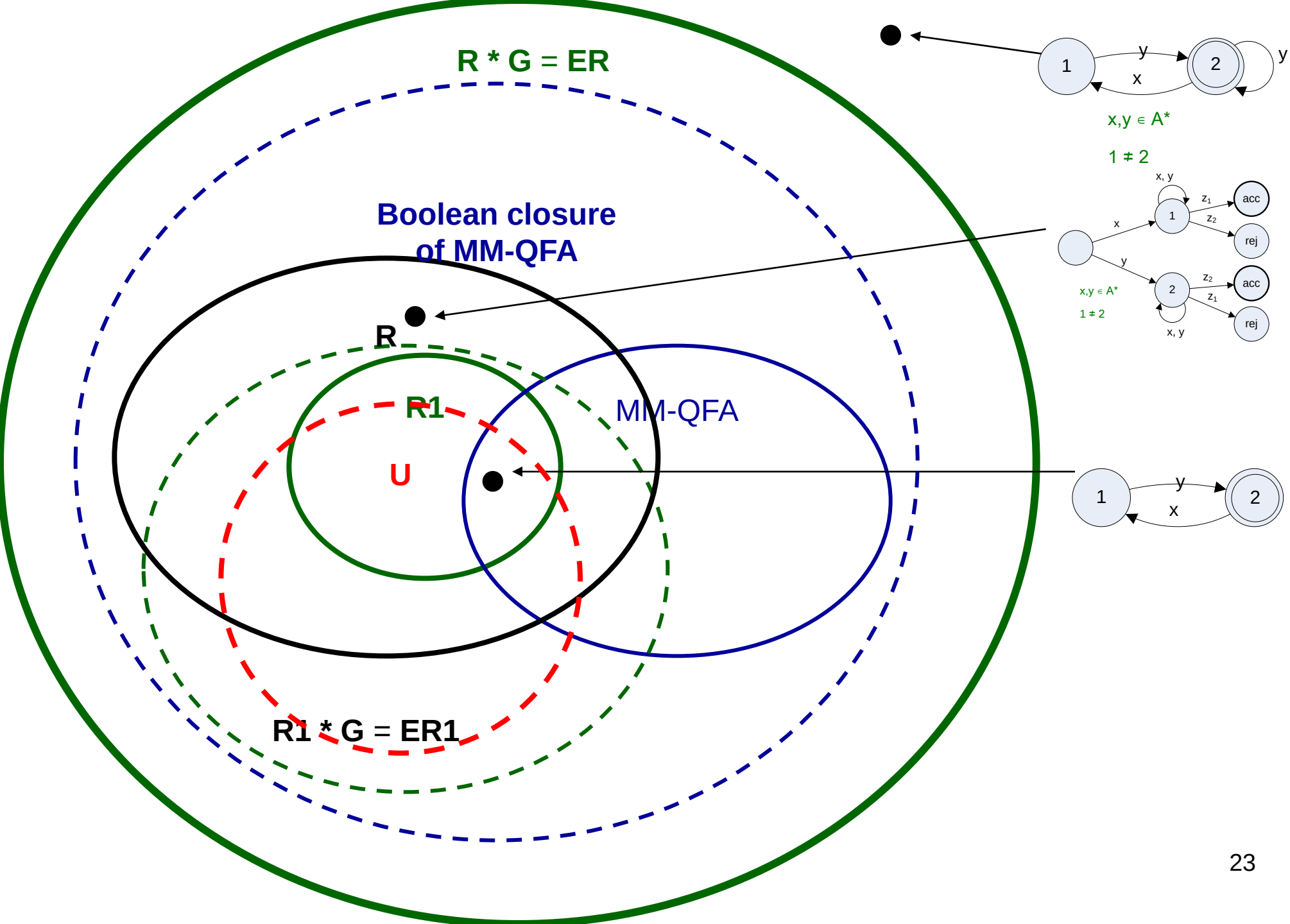
- **Theorem**: Any R_1-trivial language may be decomposed [in unique way] as a disjoint union of the languages recognised by the following automata :

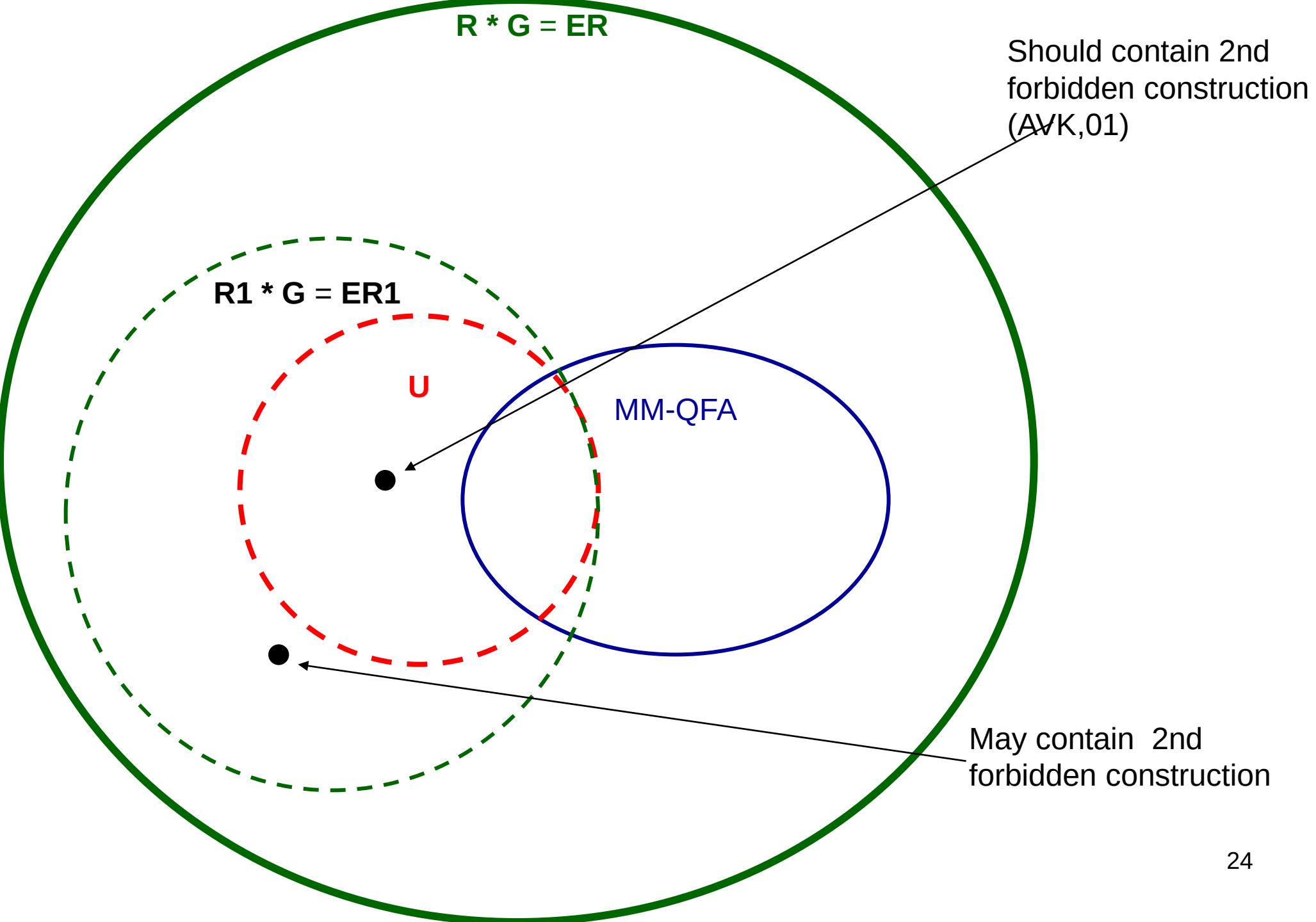


R1 trivial languages 2-2



- Any language above may be recognised by DH-PRA, MM-QFA and EQFA
- Boolean closure of the languages recognised by MM-QFA, DH-PRA and EQFA contains any R1-trivial language
- Follows from **R1** $\in$ **R**


R * G = ER
Boolean closure
of MM-QFA
R
R1
U
MM-QFA
R1 * G = ER1
1
2
y
x
y
x,y ∈ A*
1 ≠ 2
x, y
x
y
z1
z2
acc
rej
1
2
x,y ∈ A*
1 ≠ 2
x, y


R * G = ER
Should contain 2nd forbidden construction (AVK,01)
R1 * G = ER1
U
MM-QFA
May contain 2nd forbidden construction

Future Agenda

1) prove that MM-QFA generate any *R*-trivial monoid (proved);
2) characterise R_1-trivial languages recognised by MM-QFA;
3) characterise *R*-trivial languages recognised by MM-QFA;
4) prove that MM-QFA generate any monoid in **ER**;
5) characterise the languages (with syntactic monoids in **ER**) recognised by MM-QFA

Thank You!