University of Latvia
Faculty of Computing

Search by quantum walks on two dimensional grid without amplitude amplification

Andris Ambainis *Artūrs Bačkurs*
Alexander Rivosh *Nikolajs Nahimovs* *Raitis Ozols*

Supported by European Social Fund grant 2009/0216/1DP/1.1.1.2.0/09/APIA/VIAA/044

Supported by
EUROPEAN SOCIAL FUND
EUROPEAN UNION
INVESTMENT IN YOUR FUTURE

Quantum walks on 2D grid

Each step of quantum walk consists of two transformations.

1st transformation

Apply Grover's ***diffusion*** for all **unmarked** locations

$$D = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

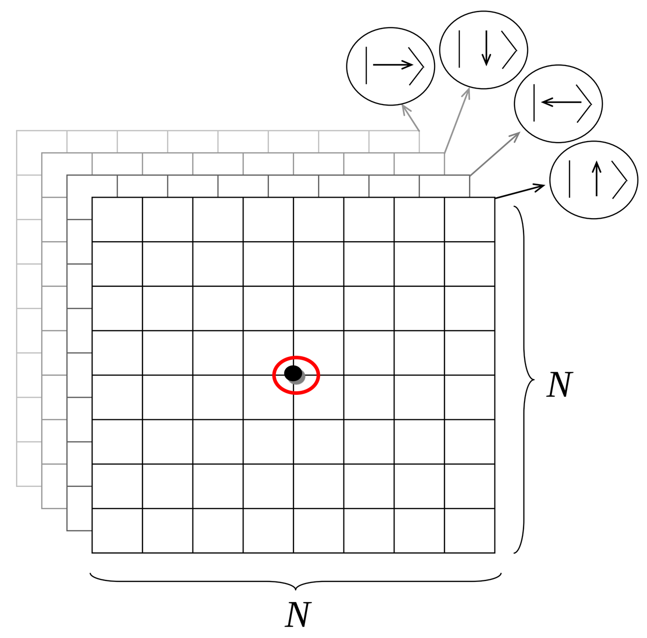
Apply *-Identity* transformation for **marked** location

$$-I = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

2nd transformation

Apply ***shift*** transformation for **all** locations in all four planes, according to the following rules:

$$S_{ff}: \quad |\rightarrow\rangle \otimes |x,y\rangle \rightarrow |\leftarrow\rangle \otimes |x+1,y\rangle$$
$$|\leftarrow\rangle \otimes |x,y\rangle \rightarrow |\rightarrow\rangle \otimes |x-1,y\rangle$$
$$|\uparrow\rangle \otimes |x,y\rangle \rightarrow |\downarrow\rangle \otimes |x,y+1\rangle$$
$$|\downarrow\rangle \otimes |x,y\rangle \rightarrow |\uparrow\rangle \otimes |x,y-1\rangle$$



Previously known results

We study a search by quantum walks on a finite two-dimensional grid according to [AKR05]. For grid of size $\sqrt{N} \times \sqrt{N}$ the original [AKR05] algorithm takes O($\sqrt{N \log N}$) steps and finds a marked location with probability O(1 / log *N*). This probability is small, thus the algorithm needs amplitude amplification to get Θ(1) probability. The amplitude amplification adds an additional O($\sqrt{\log N}$) factor to the number of steps, making it O($\sqrt{N}$ log *N*).

Main results

We show that the probability of being in O($\sqrt{N}$) neighbourhood of marked location, i.e. at O($\sqrt[4]{N}$) distance from the marked location, is Θ(1). This allows us to replace amplitude amplification with classical post processing which does not increase time complexity of the algorithm and leads to O($\sqrt{\log N}$) speed-up.

A O($\sqrt{\log N}$) speed-up was achieved by other research groups. However, their approaches to this problem are based on modification of the original algorithm [Tul08] or both the algorithm and the structure of the graph [KM+10].

Note that our approach doesn't require any modification of graph structure or original algorithm.

-State of quantum walk (almost) stays in 2-dimensional subspace;

- After O($\sqrt{N \log N}$) steps, it reaches a state in this subspace that is perpendicular to the starting state

Let $|\psi\rangle = \sum_{j=0}^{\sqrt{N}-1} \sum_{j'=0}^{\sqrt{N}-1} \sum_{d} \alpha^t_{j,j',d} |j,j',d\rangle$ be the state of the quantum walk after *t* steps

We can choose $t = O(\sqrt{N \log N})$ so that for any set $S \subseteq \{0, ..., \sqrt{N}-1\}^2$,

we have $\sum_{(j,j')\in S} |\alpha^t_{j,j',\Uparrow}|^2 \geq C^2 \sum_{(j,j')\in S} (f(j,j') - f(j-1,j'))^2 + o(1)$

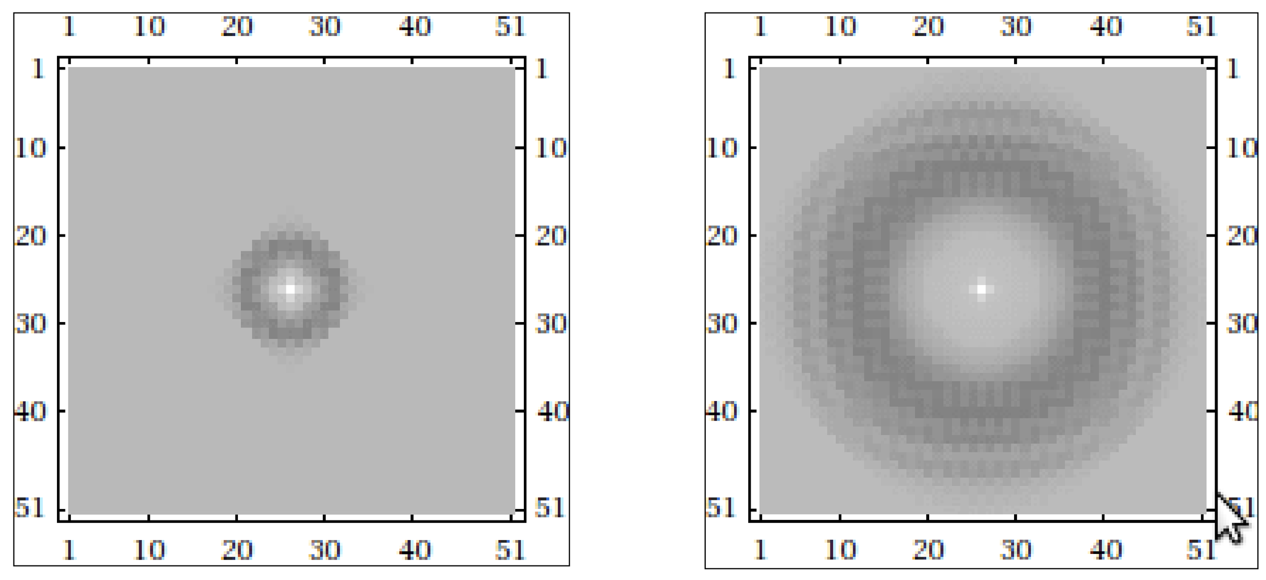
where $f(j,j') = \sum_{(k,l)\neq(0,0)} \frac{1}{2 - \cos\frac{2k\pi}{N} - \cos\frac{2l\pi}{N}} w^{kj+lj'}$ and $C = \Theta(\frac{1}{N\sqrt{\log N}})$.

- Via sequence of approximations, we obtain that the amplitude of being in location (*x*, *y*) scales as O(1/*k*), *k* = max(*x*, *y*).

- Hence, the probability of being in $N^{\varepsilon} * N^{\varepsilon}$ neighbourhood scales as Θ(ε).

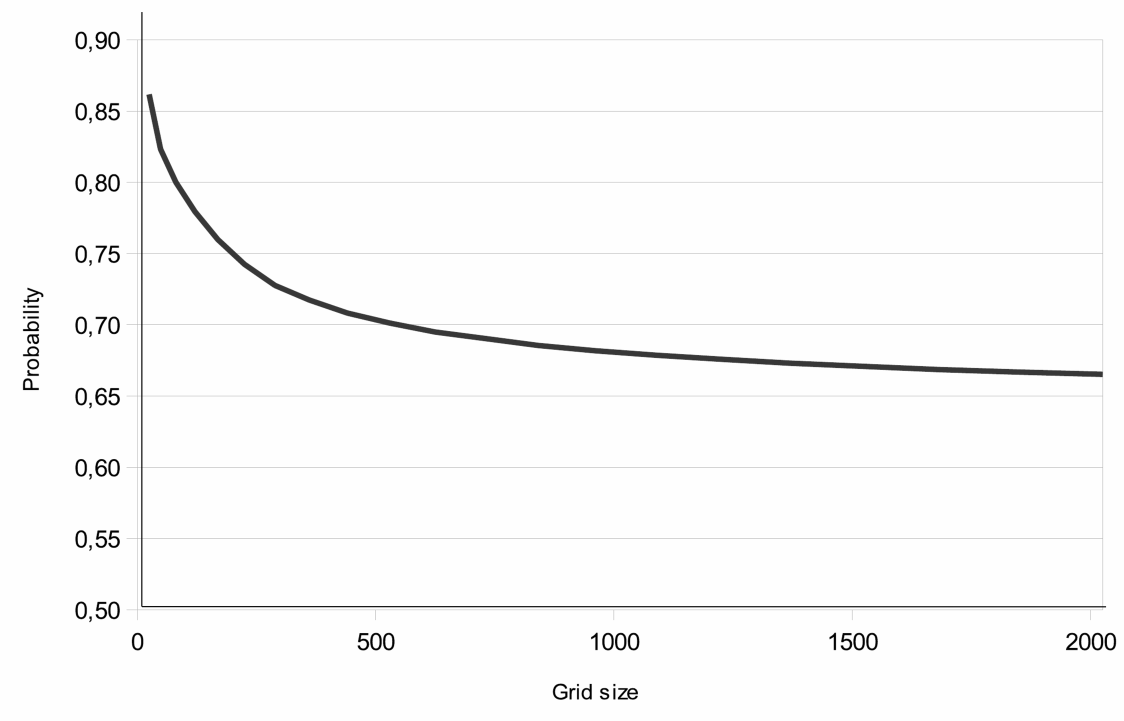
The results of simulation

Quantum walk state: at the begining and close to the end



Simulation shows that probability concentrates close to the marked location

We have measured the probability inside *R*= $\sqrt{N}$ neighbourhood of the marked location



Probability to be within $\sqrt{N}$ neibourhood from the marked location.

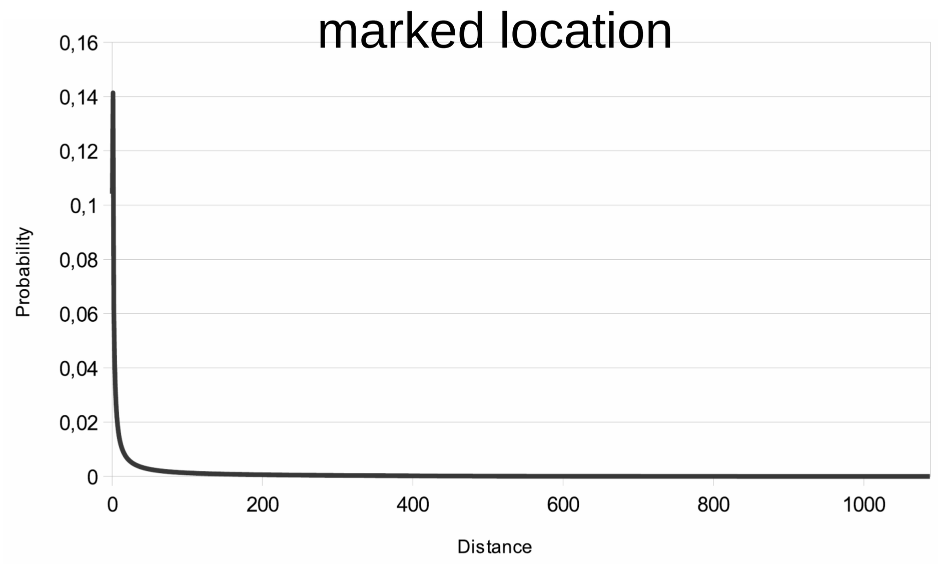
Numerical experiments showed, that the following formula holds:

$$\Pr[R = r] \approx \frac{\Pr[R = 0]}{r}$$

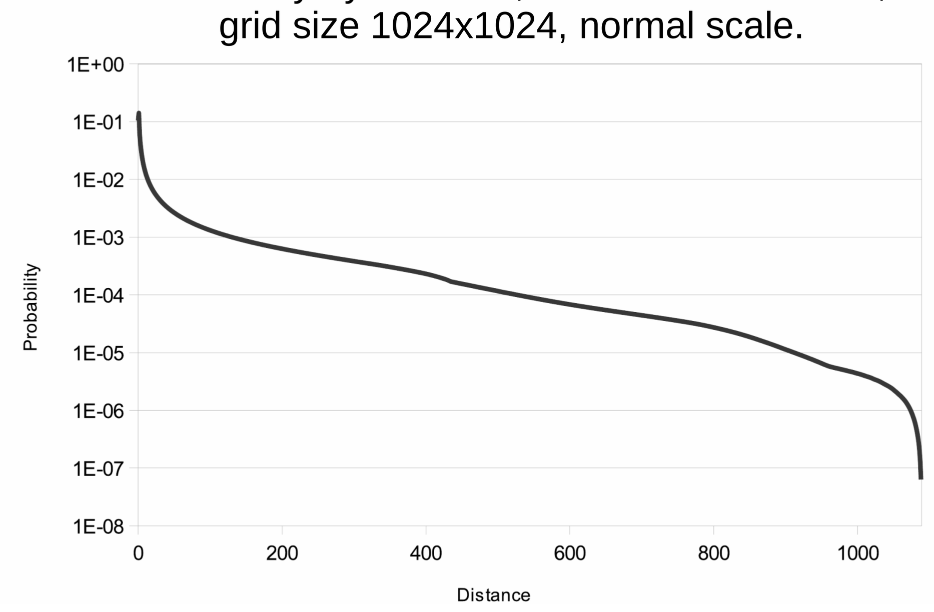
Total probability at distance *r* from the marked location

Distance from the marked location

Probability of measuring the marked solution



Probability by distance, one marked location, grid size 1024x1024, normal scale.



Probability by distance, one marked location, grid size 1024x1024, logarithmic scale.

References

- [AKR05] A. Ambainis, J. Kempe, A. Rivosh. Coins make quantum walks faster. *Proceedings of* SODA'05, 1099-1108, 2005.
- [Tul08] A. Tulsi. Faster quantum-walk algorithm for the two dimensional spatial search. *Phys. Rev. A* 78.012310, 2008
- [KM+10] H. Krovi, F. Magniez, M. Ozols, J. Roland. Finding is as easy as detecting for quantum walks. *Proceedings of* ICALP'10