

Variable time amplitude amplification and a faster quantum algorithm for systems of linear equations



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Usual amplitude amplification

[Brassard, Mosca, Tapp, 2000]

- Input: a quantum algorithm that stops after T steps, succeeds with probability p .
- Result: a quantum algorithm that succeeds with probability $\Omega(1)$, runs in time $O(T/\sqrt{p})$.
- Generalizes Grover's quantum search.

New amplitude amplification

• Input: a quantum algorithm that may stop at times T_1, T_2, \dots, T_m , succeeds with probability p .

• Result: a quantum algorithm that succeeds with probability $\Omega(1)$, runs in time $O\left(\frac{T_{av}}{\sqrt{p}} \log T_{av}\right)$

• T_{av} – average stopping time:

$$T_{av} = \sqrt{\sum_i p_i T_i^2}$$

(L_2 average).

• Generalizes variable time search of [Ambainis, STACS'2008]

HHL linear equation algorithm

• System of linear equations $A x = b$.

• A, b are given, we have to find x .

• HHL:

$$\begin{aligned} |b\rangle &= \sum_i b_i |i\rangle \\ &\xrightarrow{\quad A^{-1} \quad} \\ |x\rangle &= \sum_i x_i |i\rangle \end{aligned}$$

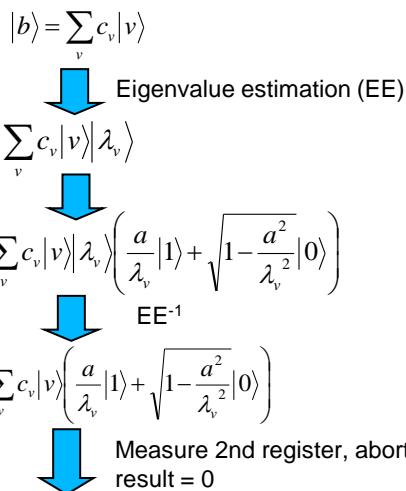
Implementing A^{-1}

Eigenvectors of A : $A|v\rangle = \lambda |v\rangle$.

$$|b\rangle = \sum_v c_v |v\rangle$$

$$|x\rangle = \sum_v c_v \lambda_v^{-1} |v\rangle$$

HHL implementation



Running time analysis:

• $\lambda_{\min}, \lambda_{\max}$ – smallest and biggest eigenvalue.

• $\kappa = \lambda_{\max}/\lambda_{\min}$.

• Assume $\lambda_{\max} = 1$.

Running time: $T = O(1/\lambda_{\min}) = O(1/\kappa)$ for eigenvalue estimation.

Success probability:

$$p \geq \left(\frac{a}{\lambda_{\max}} \right)^2 = \frac{1}{\kappa^2}$$

Apply amplitude amplification, running time:

$$O\left(\frac{T}{\sqrt{p}}\right) = O(\kappa^2)$$

Our implementation

• Run eigenvalue estimation several times, with running times 1, 2, 4, ...

• Stop when a good estimate is obtained.

• Amplify, using variable-time amplitude estimation.

• Time:

$$O\left(\frac{T_{av}}{\sqrt{p}} \log T_{av}\right) = O(\kappa \log^3 \kappa)$$

• Main idea is simple, but technical details are complicated.

• Nearly optimal: HHL show that $\Omega(\kappa^{1-o(1)})$ steps are necessary, unless BQP=PSPACE.

• Open problem: more applications for linear equations quantum algorithms.

• Our experience: dependence on condition number matters!