WEAK RELATIVE ANNIHILATORS IN POSETS

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Abstract

The notion of relative annihilator, applied to meet semilattices by J.C. Varlet and used by him to define certain relativized versions of distributivity and implicativity of a semilattice, is weakened and adapted for arbitrary posets. In terms of such annihilators, the notions of semidistributivity and weak relative pseudocomplementation, usually considered in the context of meet semilattices and lattices, are defined for posets. Necessary and sufficient conditions are given under which a weakly relatively pseudocomplemented poset is sectionally or relatively pseudocomplemented.

1 Introduction

Suppose that S is a meet semilattice (or a lattice) and $a, b \in S$. The annihilator of a relative to b is the subset $\{x\colon\ a \wedge x \leq b\}$ of S. This notion was introduced by M. Mandelker for lattices in [18] and by J.C. Varlet for meet semilattices in [28]. In the latter paper, also the notions of an a-distributive and an a-implicative semilattice (for an arbitrary element a) were introduced in terms of relative annihilators and ideals; such semilattices were further investigated by C.S. Hoo in [13, 14]. We generalize this approach and define meet semi-distributivity and weak relative pseudocomplementation in an arbitrary poset; actually, our interest in the latter operation served as a motivation for introducing the notion of weak relative annihilator.

A meet semilattice is said to be *weakly relatively pseudocomplemented* if all the maxima

$$\max\{u\colon\ u \wedge x = y\} \tag{1}$$

with $y \leq x$ exist. This concept goes back to [19], where the congruence lattice of a semilattice was shown to be weakly relatively pseudocomplemented (without using the term, which appeared later in [23]). Weak relative pseudocomplements are discussed also, for example, in [1, 8, 9, 11], [10, 21] and [24, 25, 29].

Proposition 2.1 of [23] states that a semilattice is weakly relatively pseudocomplemented if and only if it has pseudocomplemented principal filters. A systematic study of lattices and semilattices having the latter property (*sectionally pseudocomplemented (semi)lattices*) was initiated by I. Chajda [2, 3]. See also the monograph [4] and references therein for information on sectionally pseudocomplemented semilattices and lattices. Another consequence of the mentioned proposition from [23] is that the class of weakly relatively pseudocomplemented semilattices coincides with that of semilattices with 1 in which all closed intervals are pseudocomplemented; for the early study of such semilattices see [15, 16, 27].

Let S be a weakly relatively pseudocomplemented, or sectionally pseudocomplemented, semilattice. Given elements $x, y \in S$ with $y \leq x$, we denote the weak pseudocomplement of x relative to y in S (i.e. the maximum (1)) by $x * y$. The arising operation $*$ is partial; there are good reasons to consider a total binary operation extending $*$ as a kind of implication. A natural question arises how to extend $*$ in a reasonable way and when it is possible to do. Several (and different) answers to the question can be found in the literature: see [2, 3, 5, 6, 12, 22, 23, 26]. It is of some interest to investigate also purely implicational algebras based on weakly relatively pseudocomplemented posets. An example of such algebras are sectionally j-pseudocomplemented posets mentioned in [5].

For limitations of size, in this introductory paper we only state some elementary properties of weak relative annihilators and weakly relatively pseudocomplemented posets. In the next section we introduce the notion of weak relative annihilator; in combination with an appropriate notion of an ideal, weak relative annihilators provide a basis for defining meet-semidistributivity and weak relative pseudocomplementation in posets and considering connections between these. Weakly relatively pseudocomplemented posets is the subject of Section 3. In the last section (Section 4), we compare weak relative pseudocomplementation with sectional pseudocomplementation (in contrast to semilattices, the two kinds of conditioned pseudocomplementation do not, generally, agree in posets) and with relative pseudocomplementation. See the note [9] for use of the terms 'relative pseudocomplementation' and 'weak relative complementation'.

2 Weak relative annihilators

Recall that a lattice is said to be *meet-semidistributive* if, for every p an all x, y, z,

$$\text{if } x \wedge y = p = x \wedge z, \text{ then } p = x \wedge (y \vee z).$$

This condition has the following $\vee$-free consequence:

$$\text{if } x \wedge y = p = x \wedge z, \text{ then } p = x \wedge k \text{ for some } k \geq y, z \tag{2}$$

and is, in fact, equivalent to it (cf. [7]). Indeed, if $x \wedge y = p = x \wedge z$, then $p \leq x, y, z$ and, hence, $p \leq x \wedge (y \vee z)$ and $p \leq x \wedge k$ as well whenever $k \geq x, y$. On the other hand, the consequent of (2) implies that $y \vee z \leq k$ and $x \wedge (y \vee z) \leq x \wedge k \leq p$ for some k. Eventually, we can already apply the notion of semidistributivity to arbitrary meet semilattices.

Now let P be any poset. By a *weak relative annihilator* (or *wr-annihilator*, for short) we mean a nonempty subset $\{u\colon (u] \cap (a] = (b]\}$ of P, where $a, b \in P$. Note that $(u] \cap (a] = (b]$ if and only if the l.u.b. of u and a exists and equals to b. Therefore, such a subset exists for some a, b if and only if $a \geq b$, and we then call it the *weak annihilator of a relative to b* and denote by $\langle a, b \rangle$.

Proposition 1. *Weak relative annihilators have the following properties (*1 *stands for the greatest element of the poset, if it exists):*

(a) $\langle a, a\rangle = [a)$,

(b) $\langle a, b\rangle \subseteq [b)$,

(c) $b \in \langle a, b\rangle$,

(d) $\langle 1, b\rangle = \{b\}$,

(e) *if* $c \in \langle a, b\rangle$, *then* $[b, c] \subseteq \langle a, b\rangle$,

(f) *if* $c \in [b, a]$, *then* $\langle a, b\rangle \subseteq \langle c, b\rangle$.

(g) *if* $c \in \langle a, b\rangle$ *and* $c \leq a$, *then* $c = b$.

A *Varlet ideal*, or just *ideal*, of a poset P is a non-empty hereditary up-directed subset I of S: $I \neq \varnothing$, if $x \in I$ and $y \leq x$, then $y \in I$, and if $x, y \in I$, then $x, y \leq z$ for some $z \in I$. Of course, every principal order ideal is an ideal, and, if P is down-directed, the set $\mathcal{I}$ of all ideals is a lattice, where meet coincides with intersection.

We call ideals of the subposet $[b)$ *b-ideals* of P and denote the set of such ideals by $\mathcal{I}_b$. Notice that a principal b-ideal is actually a segment $[b, a]$ with $a \geq b$, and conversely. By a *relative ideal* of P we mean any of its b-ideals with arbitrary b.

We now say that a poset P is *m-semidistributive at p* ('m' for 'meet') if every wr-annihilator $\langle x, p\rangle$ is a p-ideal, and that P is *m-semidistributive* if it is m-semidistributive at every $p \in P$. It is easily seen that in meet semilattices this definition reduces to (2) (but it is weaker than those introduced for posets in [7] and [17]).

Recall that, in a poset with the least element, the pseudocomplement x^* of an element x is the greatest element that has a single common lower bound with x

Theorem 2. *Suppose that P is a poset m-semidistributive at b. Then the annihilator $\langle a, b\rangle$ is the pseudocomplement of the interval $[b, a]$ in $\mathcal{I}_b$.*

Proof. The supposition implies that $\langle a, b\rangle$ is a b-ideal. We have to prove that $\langle a, b\rangle$ is the greatest one among those b-ideals J that satisfy the condition $J \cap [b, a] = \{b\}$. At first, $\langle a, b\rangle$ is one of such b-ideals, for

$$((u] \cap (a] = (b] \text{ and } b \leq u \leq a) \text{ iff } u = b.$$

Further, every such a b-ideal J is a subset of $\langle a, b\rangle$: if $u \in J$ (then $u \geq b$) and v is a lower bound of u and a in $[b)$, then v belongs both to J and $[b, a]$; hence, $v = b$. $\square$

We say that P is *tight at y* if, for all $u, x \geq y$, the element y is the meet of u and x in P if and only if it is their meet in $[y)$ (the 'only if' direction is, in fact, trivial). As direct checking shows, this is the case if and only if, for all u, x,

$$y \leq u, x \text{ implies that } [y, u] \cap [y, x] = \{y\} \text{ iff } (u] \cap (x] \subseteq (y]. \tag{3}$$

It is shown in the proof of Lemma 1 in [5] that in a join semilattice the condition (3) is fulfilled for all y, u and x. This holds true also for meet semilattices.

Proposition 3. *A meet semilattice is tight at all of its elements.*

Proof. Let S be a meet semilattice. Assume that $y \leq u, x$, and suppose that $y = u \wedge x$. If $y \leq v \leq u, x$, then $v = u \wedge x = y$. Conversely, suppose that y is the meet of u and x in $[y)$, hence, the single lower bound of u and x belonging to $[y)$. Then $y \leq u \wedge x \leq u, x$, so that $y = u \wedge x$. $\square$

We say that the poset P *has enough meets* if it is tight at every y. Therefore, both join and meet semilattices have enough meets.

The next theorem is a counterpart of Theorem 4.2 in [28], which is stated for meet semilattices and their ideal latices.

Theorem 4. *Suppose that P is a poset tight at p. Then P is m-semidistributive at p if and only if the lattice $\mathcal{I}_p$ is pseudocomplemented.*

Proof. Let $\mathcal{I}_p$ be pseudocomplemented, and let I^* stand for the pseudocomplement of arbitrary $I \in \mathcal{I}_p$; we have to prove that then every annihilator $\langle x, p\rangle$ is a p-ideal. By Proposition 1(b,e), it is a hereditary subset of $[p)$. If $u \in \langle x, p\rangle$, then $[p, u] \cap [p, x] = \{p\}$ and, further, $[p, u] \subseteq [p, x]^*$. Likewise, $v \in \langle x, p\rangle$ implies that $[p, v] \subseteq [p, x]^*$; in particular, $u, v \in [p, x]^*$. As $[p, x]^*$ is an ideal from $\mathcal{I}_p$, now $[p, k] \subseteq [p, x]^*$ for some $k \geq u, v$. It follows by (3) that $[p, k] \cap [p, x] = \{p\}$, whence $k \in \langle x, p\rangle$.

Now assume that P is m-distributive at p, and suppose that $I \in \mathcal{I}_p$. Let J be the intersection of all annihilators $\langle x, p\rangle$ with $x \in I$. Clearly, J is nonempty and, hence, is a p-ideal. Moreover, J is the pseudocomplement of I: as $p \in K \cap I$,

$$\begin{aligned} K \subseteq J &\Leftrightarrow \forall(u \in K)\forall(x \in I) u \in \langle x, p\rangle \\ &\Leftrightarrow \forall(u \in K)\forall(x \in I)(u] \cap (x] = (p] \\ &\Leftrightarrow \forall(u \in K)\forall(x \in I)([p, u] \cap [p, x] = \{p\} \\ &\Leftrightarrow K \cap I = \{p\}; \end{aligned}$$

we once more used (3). $\square$

3 From annihilators to weak relative pseudocomplementation

Suppose that $x, y \in P$ and $y \leq x$. A *weak pseudocomplement of x relative to y* is the element z defined by

$$z := \max\langle x, y\rangle = \max\{u\colon\ (u] \cap (x] = (y]\} = \max\{u \geq y\colon\ (u] \cap (x] \subseteq (y]\}. \quad (4)$$

More explicitly,

$$\text{if } y \leq x, \text{ then, for all } u \geq y, \ (u \leq z \text{ if and only if } (u] \cap (x] \subseteq (y]). \quad (5)$$

A poset is said to be *weakly relatively pseudocomplemented at y* (or just *wr-pseudocomplemented at y*, for short) if all weak pseudocomplements (wr-pseudocomplements) relative to y exist, and it is *weakly relatively pseudocomplemented* (*wr-pseudocomplemented*) if it is wr-pseudocomplemented at every y. If $x * y$ stands for the wr-pseudocomplement of x relative to y, then the (partial) operation $*$ defined in this way is called *weak relative pseudocomplementation*, or just *wr-pseudocomplementation*.

Therefore, a poset is wr-pseudocomplemented if and only if all wr-annihilators in it are principal ideals. In particular, every such a poset is m-semidistributive. Recall that if P satisfies the Ascending Chain Condition, then every nonempty subset of P has a maximal element. Hence, all of its relative ideals in $\mathcal{I}_p$ are principal; consequently, if P is, in addition, meet-semidistributive, then it is wr-pseudocomplemented.

The subsequent lemma provides an axiomatic characteristic of wr-pseudocomplementation.

Lemma 5. *A poset P with an additional partial operation $*$ is a wr-pseudocomplemented poset if and only if the following holds in P:*

(a) *$x * y$ is defined if and only if $y \leq x$,*

(b) *if $y \leq x$ and $v \leq x * y, x$, then $v \leq y$,*

(c) *if y is the greatest lower bound of u and x, then $u \leq x * y$.*

Proof. If $y \leq x$, then, according to (4), z is the weak pseudocomplement of x relative to y if and only if, for all $v, u \in P$,

(i) $v \leq z, x$ if and only if $v \leq y$, and
(ii) if y is the greatest lower bound of u and x, then $u \leq z$.

It follows from (ii) and the supposition $y \leq x$ that $y \leq z$, so that "if" part of (i) is easily derivable. It is now obvious that (a), (b) and (c) hold whenever $x * y$ is the weak pseudocomplement of x relatively to y. On the other hand, if the operation $*$ satisfies the conditions (a), (b) and (c), then it has the domain appropriate for weak relative pseudocomplementation and satisfies also (4). $\square$

We now list a few elementary properties of weak relative pseudocomplementation.

Proposition 6. *The operation $*$ has the following properties:*

$(*_1)$: *if $y \leq x$, then $y \leq x * y$,*
$(*_2)$: *if $y \leq x$, then $x \leq y * y$,*
$(*_3)$: *if $y \leq x$, then $(y * y) * x = x$,*
$(*_4)$: *if $z \leq x, y$ and $x \leq y * z$, then $y \leq x * z$,*
$(*_5)$: *if $y \leq x$, then $x \leq (x * y) * y$,*
$(*_6)$: *if $z \leq y \leq x$, then $x * z \leq y * z$,*
$(*_7)$: *if $y \leq v \leq x, x * y$, then $v = y$,*

Proof. $(*_1)$ and $(*_2)$ are particular cases of Lemma 5(c). One half of $(*_3)$ follows from $(*_1)$ in virtue of $(*_2)$; the reverse inequality is obtained from the particular case of Lemma 5(b) with $x := y * y$, $y := x$ and $v := (y * y) * x$ (we have $x \leq y * y$ by $(*_2)$). To prove $(*_4)$, first observe that $v \leq z$ whenever $z \leq y$ and $v \leq y, y * z$ (Lemma 5(b)). Then $z \leq x, y$ and $x \leq y * z$ imply that z is the g.l.b. of x and y, and, by Lemma 5(c), that $y \leq x * z$. In virtue of $(*_1)$, the condition $(*_4)$ implies $(*_5)$ and, further, also $(*_6)$. Condition $(*_7)$ follows directly from Lemma 5(c). □

Therefore, according to $(*_1)$, every upper end $[b)$ of a poset is closed under $*$. It follows from $(*_2)$ that, in a wr-pseudocomplemented poset P $y * y = \max[y)$. So, if b is a maximal element of P, then $b = b * b$. Hence, if P is up-directed, then it must have the greatest element 1; then $(*_3)$ reduces to the simple equality $1 * x = x$. Note that a down-directed wr-pseudocomplemented poset is always also up-directed: if $z \leq x, y$ then $x, y \leq z * z$. Condition $(*_4)$ shows that weak pseudocomplementation relative to z is a symmetric Galois connection in $[z)$.

4 Two other kinds of local pseudocomplementation

A poset P is said to be *sectionally pseudocomplemented* if every its upper section $[p)$ is pseudocomplemented. If x and y are elements of P with $x \geq y$, then the pseudocomplement of x in the upper section $[y)$ is an element z such that

$$z = \max\{u \geq y\colon\ [y, u] \cap [y, x] = \{y\}\}. \tag{6}$$

More explicitly,

$$\text{if } y \leq x, \text{ then, for all } u \geq y, \ (u \leq z \text{ if and only if } [y, u] \cap [y, x] = \{y\}). \tag{7}$$

The subsequent lemma is a counterpart of Lemma 5 and is proved similarly. We assume in it that the pseudocomplement of x in $[y)$ is denoted by $x * y$, so that sectional pseudocomplementation is treated as a partial binary operation on P. This local stipulation will not infer with the notation for wr-complements assumed in the introduction and the previous section.

Lemma 7. *A poset P with an additional partial operation $*$ is sectionally pseudocomplemented if and only if the following holds in P:*

(a) *$x * y$ is defined if and only if $y \leq x$,*

(b) *if $y \leq v \leq x * y, x$, then $v \leq y$,*

(c) *if y is a maximal lower bound of u and x, then $u \leq x * y$.*

Now (5), (7) (3) lead us to the following conclusion.

Proposition 8. *Let P be a poset, and let x, y be elements of P such that $x \geq y$. If y is tight, then z is the weak pseudocomplement of x relatively to y if and only if z is the pseudocomplement of x in $[y)$.*

Corollary 9. *A poset which has enough meets is wr-pseudocomplemented if and only if it is sectionally pseudocomplemented.*

Let us now compare the concept of wr-pseudocomplementation with that of relative pseudocomplementation. The latter is well-known in lattices and applies without changes to meet semilattices: z is the *pseudocomplement of x relative to y* if $z = \max\{u\colon\ u \wedge x \leq y\}$. Relative pseudocomplementation has been defined also for arbitrary posets P:

$$z := \max\{u\colon\ (u] \cap (x] \subseteq (y]\}; \tag{8}$$

more explicitly, for every $u \in P$,

$$u \leq z \text{ if and only if } (u] \cap (x] \subseteq (y]$$

(cf. (5)). To proceed, we need likewise to adapt to posets also the notion of (meet-) modularity. Namely, we say that a poset P is *modular* if, for all u, x, y,

$$(u] \cap (x] \subseteq (y] \text{ implies that } (u'] \cap (x] = (y] \text{ for some } u' \geq u.$$

We call an arbitrary element y of P *modular* if it satisfies the displayed condition for every u and x. This definition reduces to the standard one if P is a meet semilattice.

Theorem 10. *Let P be a poset, and let x, y be elements of P such that $x \geq y$. Then an element of P is the pseudocomplement of x relative to y if and only if it is the weak relative pseudocomplement and y is modular.*

Proof. Assume that $x \geq y$ and that z is any element of P. First suppose that z satisfies (8). As then $y \leq z$ and $(z] \cap (x] \subseteq (y]$, we conclude that $(z] \cap (x] = (y]$. If $(u] \cap (x] = (y]$, then $u \leq z$. Therefore, the first equality in (4) holds, and z is the weak pseudocomplement of x relative to y. (Note that, however, z need not be the pseudocomplement of x in $[y)$ if there are not enough meets in P.) Next suppose that $(u] \cap (x] \subseteq (y]$ for some u. It follows from (4) that $(z] \cap (x] = (y]$. As $u \leq z$, the element y turns out to be modular.

To prove the converse, suppose that (4) is fulfilled for some x, y with $y \leq x$ and that y is modular. Clearly, then $(z] \cap (x] = (y]$. If furthermore, $(u] \cap (x] \subseteq (y]$, then there is $u' \geq u$ such that $y = u' \wedge x$. We conclude that $u \leq u' \leq z$. Thus (8) is fulfilled, and z is the pseudocomplement of x relatively to y. □

Corollary 11. *A poset is relatively pseudocomplemented if and only if it modular and weakly relatively pseudocomplemented.*

We note here that, according to [27, Theorem 2], a lattice with sectionally pseudocomplemented intervals, i.e., a wr-pseudocomplemented lattice (see Introduction) is distributive if and only if it is modular. Relatively pseudocomplemented posets were rediscovered in [20] under the name ‘implicative poset’. Theorem 4 of [20] states that an implicative poset is (meet-)distributive in the following sense: for all u, x, y,

$$\text{if } (u] \cap (x] \subseteq (y],\ \text{ then } (u'] \cap (x'] = (y] \text{ for some } u' \geq u \text{ and } x \geq x'.$$

Clearly, a distributive poset is modular. We now come to the following counterpart of the mentioned theorem from [27].

Corollary 12. *A wr-pseudocomplemented poset is distributive if and only if it is modular.*

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