Jaunu čukstošās galerijas modu mikrorezonatoru izstrāde optisko frekvenču standartu un biosensoru pielietojumiem, un to raksturošana ar femtosekunžu optisko frekvenču ķemmi

Otrā ceturkšņa atskaite

31.08.2017.

NACIONĀLAIS
ATTĪSTĪBAS
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EIROPAS SAVIENĪBA
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attīstības fonds

IEGULDĪJUMS TAVĀ NĀKOTNĒ

ERAF projekts Nr. 1.1.1.1/16/A/259

Saturs

1. Projekta vadīšana un publicitāte
2. Eksperimenti
3. Modelēšana
4. Biosensori

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IEGULDĪJUMS TAVĀ NĀKOTNĒ
ERAF projekts Nr. 1.1.1.1/16/A/259

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Otrā ceturkšņa atskaite

1. Projekta vadīšana un publicitāte

NACIONĀLAIS
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IEGULDĪJUMS TAVĀ NĀKOTNĒ

ERAF projekts Nr. 1.1.1.1/16/A/259

Par projektu

- **Projekta nosaukums:** Jaunu čukstošās galerijas modu mikrorezonatoru izstrāde optisko frekvenču standartu un biosensoru pielietojumiem, un to raksturošana ar femtosekunžu optisko frekvenču ķemmi.
- **Projekta numurs:** 1.1.1.1/16/A/259
- **Projekta mērķis:** jaunu zināšanu-zinātības iegūšana CGM rezonatoru izstrādē, stabilizēšanā un modelēšanā, un rezonatoru izmantošanā biomolekulu detektēšanai, tādējādi atbalstot Latvijas Viedās specializācijas mērķu sasniegšanu, zinātnes un tehnoloģiju cilvēkkapitāla attīstību un jaunu zināšanu radīšanu tautsaimniecības konkurētspējas uzlabošanai.

•**Projekta vadītājs:** J. Alnis

•**Projekta administratīvais vadītājs:** I. Brice

•Projektu realizē LU ASI kvantu optikas laboratorija

- **Plānotie projekta galvenie rezultāti:** 4 publikācijas, 3 zinātību apraksts, 1 licences līgums.
- Paredzēti 9 konferenču apmeklējumi un 6 zinātniskās vizītes
- **Projekta īstenošanas laiks:** 01.03.2017. - 30.08.2019.

Projekta budžets

- Projekta kopējās izmaksas: 648 252,61 EUR, to skaitā ERAF finansējums (85%) - 551 014,72 EUR.
- Izdevumi pārskata periodā (tiešās izmaksas 01.06.17.- 31.08.2017):
  - Darbinieku algas 34279,88 EUR
  - Komandējumi 160 EUR
  - Datorprogrammatūras iegāde 3 133.90 EUR
- Kopā 46 967.37 EUR (tai skaitā netiešās izmaksas 9 393.59 EUR)

Iepirkumi

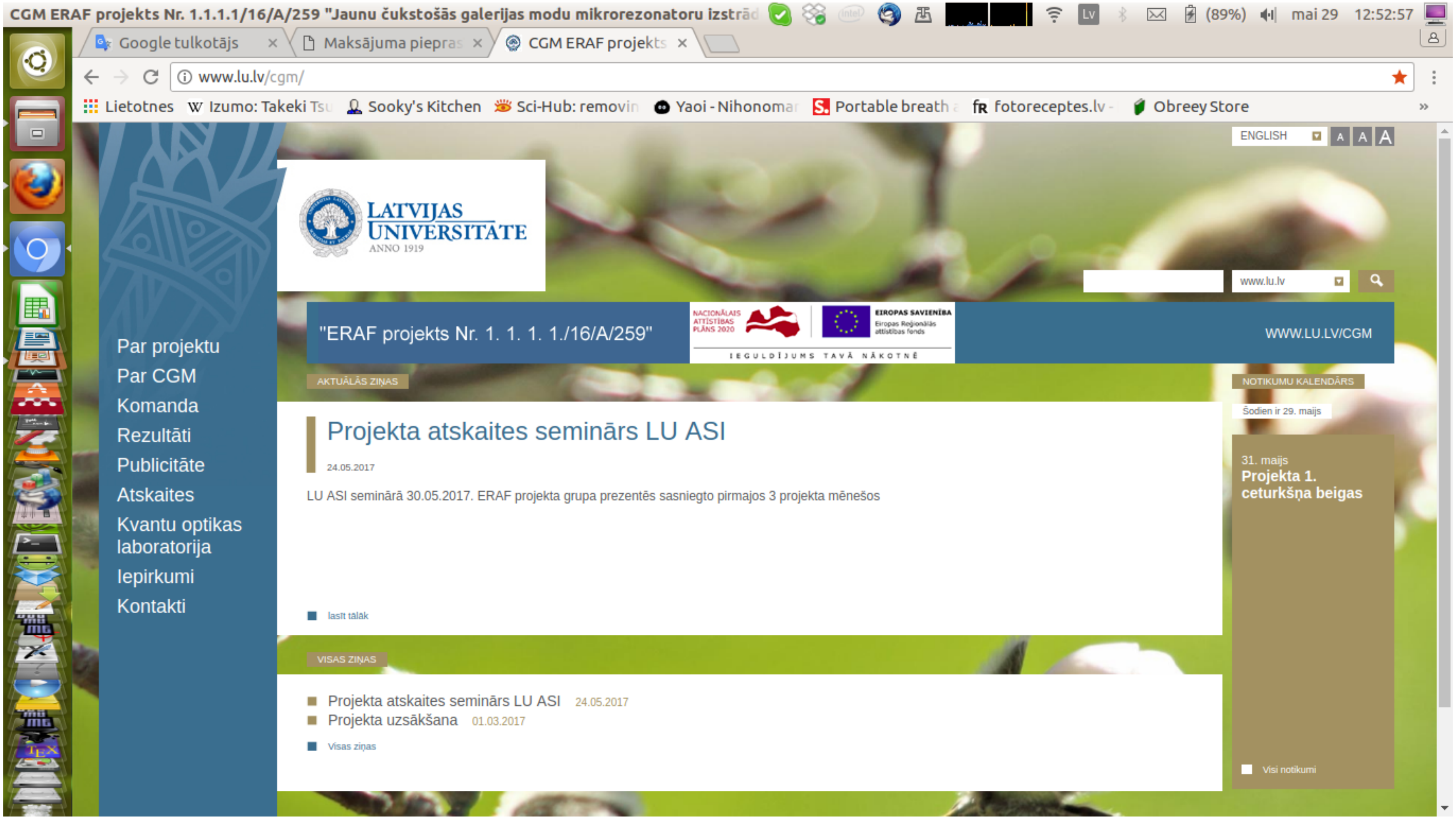
- Comsol datorprogrammas iepirkums (izpildīts).
- Materiāli iegādes iepirkums (procesā).
- Instrumentu nomas iepirkums (procesā).

Darbinieki

- Vadošie pētnieki
  - J. Alnis
  - A. Atvars
  - R. Viter
- Zinātniskie asistenti
  - I. Brice
- Dabaszinātņu laboranti
  - K. Grundšteins
  - A. Pirktiņa
  - A. A. Ūbele

Projekta dalībnieku kopbilde 2017. gada aprīlis.

Mājas lapa
www.lu.lv/cgm/



"ERAF projekts Nr. 1. 1. 1. 1./16/A/259"

- Par projektu
- Par CGM
- Komanda
- Rezultāti
- Publicitāte
- Atskaites
- Kvantu optikas laboratorija
- Iepirkumi
- Kontakti

AKTUĀLĀS ZIŅAS

Projekta atskaites seminārs LU ASI

24.05.2017

LU ASI seminārā 30.05.2017. ERAF projekta grupa prezentēs sasniegto pirmajos 3 projekta mēnešos

■ lasīt tālāk

VISAS ZIŅAS

- Projekta atskaites seminārs LU ASI 24.05.2017
- Projekta uzsākšana 01.03.2017

■ Visas ziņas

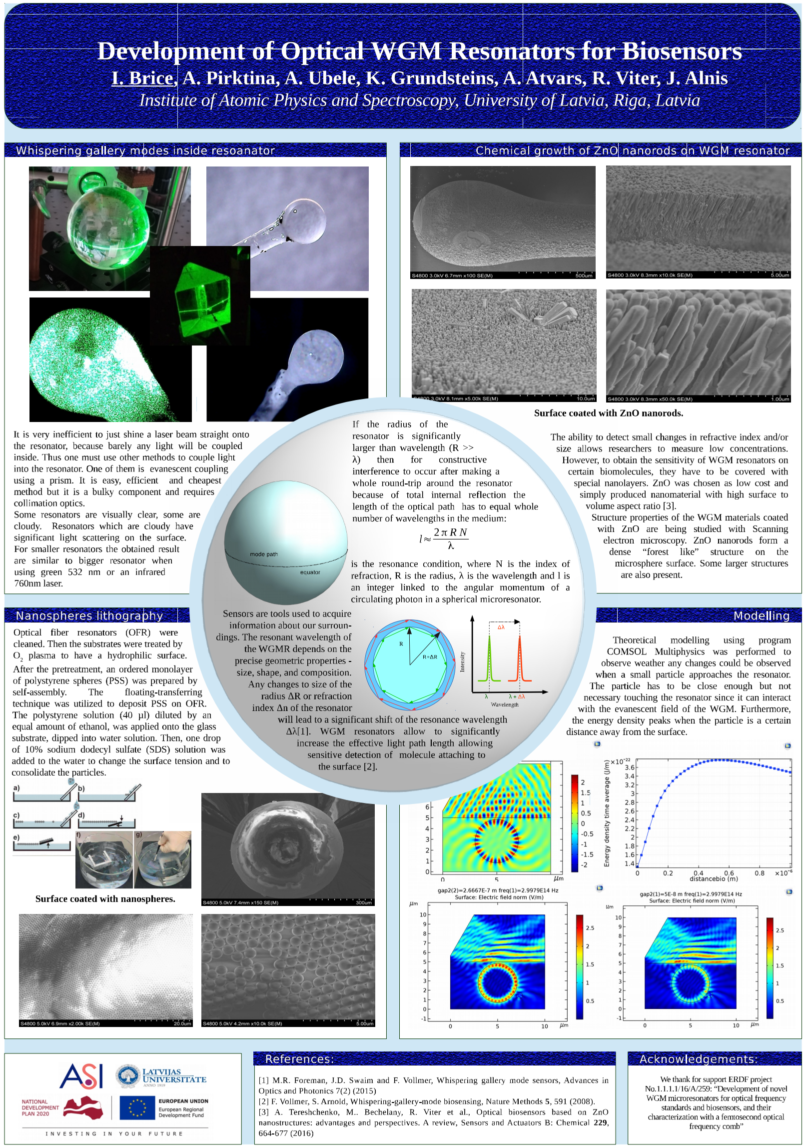
NOTIKUMU KALENDĀRS

Šodien ir 29. maijs

31. maijs
Projekta 1. ceturkšņa beigas

☐ Visi notikumi

Publicitāte



Development of Optical WGM Resonators for Biosensors

I. Brice, A. Pirktina, A. Ubele, K. Grundsteins, A. Atvars, R. Viter, J. Alnis
Institute of Atomic Physics and Spectroscopy, University of Latvia, Riga, Latvia

Whispering gallery modes inside resoanator

It is very inefficient to just shine a laser beam straight onto the resonator, because barely any light will be coupled inside. Thus one must use other methods to couple light into the resonator. One of them is evanescent coupling using a prism. It is easy, efficient and cheapest method but it is a bulky component and requires collimation optics.

Some resonators are visually clear, some are cloudy. Resonators which are cloudy have significant light scattering on the surface. For smaller resonators the obtained result are similar to bigger resonator when using green 532 nm or an infrared 760nm laser.

If the radius of the resonator is significantly larger than wavelength (R >> λ) then for constructive interference to occur after making a whole round-trip around the resonator because of total internal reflection the length of the optical path has to equal whole number of wavelengths in the medium:

$$l \approx \frac{2\pi R N}{\lambda}$$

is the resonance condition, where N is the index of refraction, R is the radius, λ is the wavelength and l is an integer linked to the angular momentum of a circulating photon in a spherical microresonator.

Sensors are tools used to acquire information about our surroundings. The resonant wavelength of the WGMR depends on the precise geometric properties - size, shape, and composition. Any changes to size of the radius ΔR or refraction index Δn of the resonator will lead to a significant shift of the resonance wavelength Δλ[1]. WGM resonators allow to significantly increase the effective light path length allowing sensitive detection of molecule attaching to the surface [2].

Chemical growth of ZnO nanorods on WGM resonator

Surface coated with ZnO nanorods.

The ability to detect small changes in refractive index and/or size allows researchers to measure low concentrations. However, to obtain the sensitivity of WGM resonators on certain biomolecules, they have to be covered with special nanolayers. ZnO was chosen as low cost and simply produced nanomaterial with high surface to volume aspect ratio [3].

Structure properties of the WGM materials coated with ZnO are being studied with Scanning electron microscopy. ZnO nanorods form a dense "forest like" structure on the microsphere surface. Some larger structures are also present.

Nanospheres lithography

Optical fiber resonators (OFR) were cleaned. Then the substrates were treated by O_2 plasma to have a hydrophilic surface.

After the pretreatment, an ordered monolayer of polystyrene spheres (PSS) was prepared by self-assembly. The floating-transferring technique was utilized to deposit PSS on OFR. The polystyrene solution (40 µl) diluted by an equal amount of ethanol, was applied onto the glass substrate, dipped into water solution. Then, one drop of 10% sodium dodecyl sulfate (SDS) solution was added to the water to change the surface tension and to consolidate the particles.

Surface coated with nanospheres.

Modelling

Theoretical modelling using program COMSOL Multiphysics was performed to observe weather any changes could be observed when a small particle approaches the resonator. The particle has to be close enough but not necessary touching the resonator since it can interact with the evanescent field of the WGM. Furthermore, the energy density peaks when the particle is a certain distance away from the surface.

References:

[1] M.R. Foreman, J.D. Swaim and F. Vollmer, Whispering gallery mode sensors, Advances in Optics and Photonics 7(2) (2015)
[2] F. Vollmer, S. Arnold, Whispering-gallery-mode biosensing, Nature Methods **5**, 591 (2008).
[3] A. Tereshchenko, M.. Bechelany, R. Viter et al., Optical biosensors based on ZnO nanostructures: advantages and perspectives. A review, Sensors and Actuators B: Chemical **229**, 664-677 (2016)

Acknowledgements:

We thank for support ERDF project No.1.1.1.1/16/A/259: "Development of novel WGM microresonators for optical frequency standards and biosensors, and their characterization with a femtosecond optical frequency comb"

Dalība konferencē "Biophotonics Riga 2017"

Jaunu čukstošās galerijas modu mikrorezonatoru izstrāde optisko frekvenču standartu un biosensoru pielietojumiem, un to raksturošana ar femtosekunžu optisko frekvenču ķemmi

Otrā ceturkšņa atskaite

2. Eksperimenti

NACIONĀLAIS ATTĪSTĪBAS PLĀNS 2020

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IEGULDĪJUMS TAVĀ NĀKOTNĒ

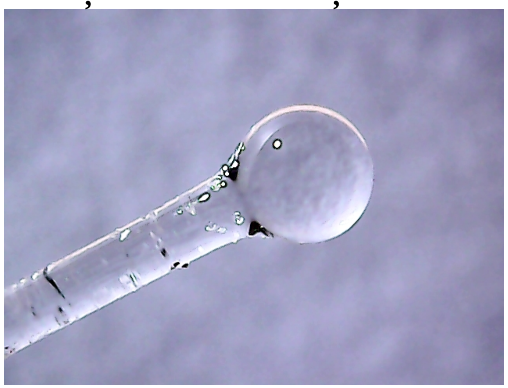
ERAF projekts Nr. 1.1.1.1/16/A/259

Ūdeņraža skābekļa liesma ČGMR un trapecveida šķiedras izgatavošanai

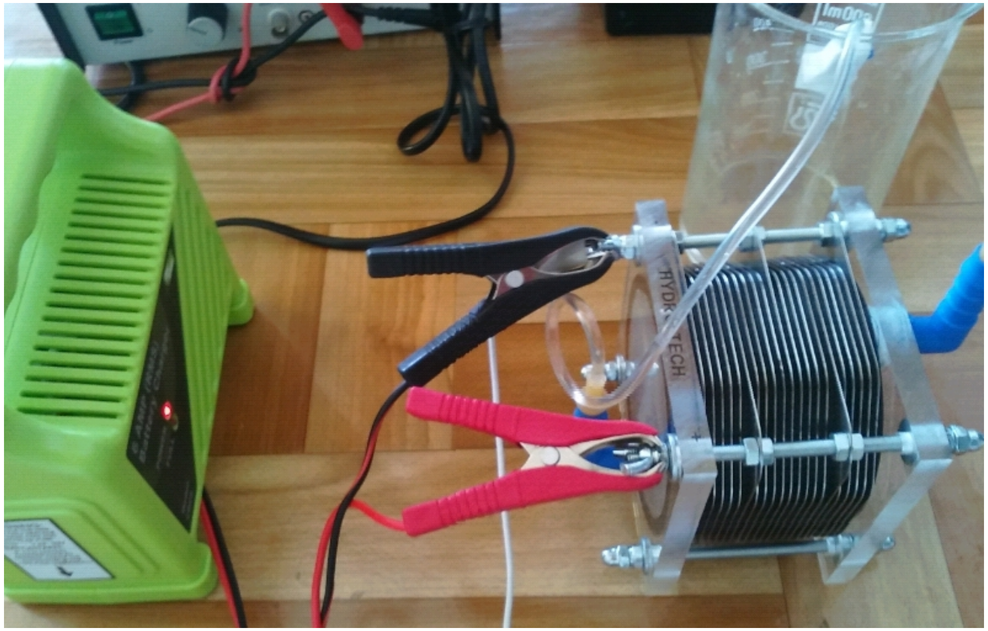
Ūdeņraža-skābekļa liesma ļauj iegūt augstas temperatūras un nerada kvēpus, kas var piesārņot optiskos materiālus. Viens veids ir izmantot saspiestas gāzes, otrs elektrolīzi. Šādu liesmu izmanto juvelieri, jo ļauj izstrādājumus lokalizēti karsēt.

Rezonators uzkausēts propāna-skābekļa liesmā

Rezonators uzkausēts ūdeņraža-skābekļa liesmā



H_2 un O_2 elektrolīzes aparātā vienas šūnas spriegums ir ap 2 V. Lai varētu darbināt no 12 V, tad 6 šūnas tiek saslēgtas virknē. Savukārt, lai iegūtu lielāku laukumu, 4 grupas tiek saslēgtas paralēli. Elektrolīzes šūnu izmanto automašīnās tas sauktas HHO gāzes ģenerācijai, kas uzlabo sadegšanas ātrumu dzinējā.



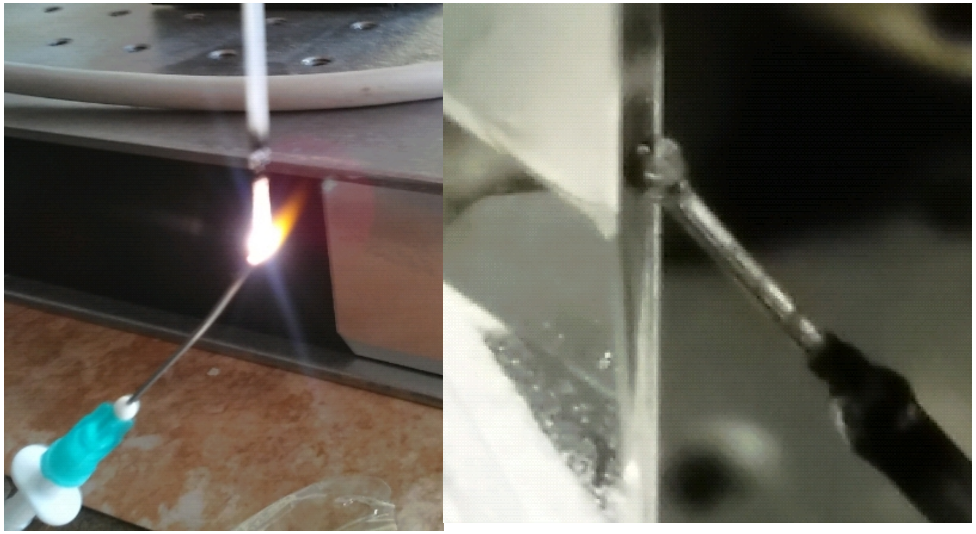
Trubiņas no aptiekas pārliešanas sistēmas. Deglis ir dažādu diametru infūziju adatas.

Liesmas degšanas ātrums ap 30 m/s. Ja gāzes plūsma mazāka, liesmu ierauj trubiņā un notiek sprādziens. Lai aizkavētu sprādzienu izmanto burbuļtrauku kā arī trauku pilnu ar smalku metāla vati. Jācenšas uzkrāt pēc iespējas mazāks sprāgstošās gāzes tilpums.

Liesma infūziju adatas galā

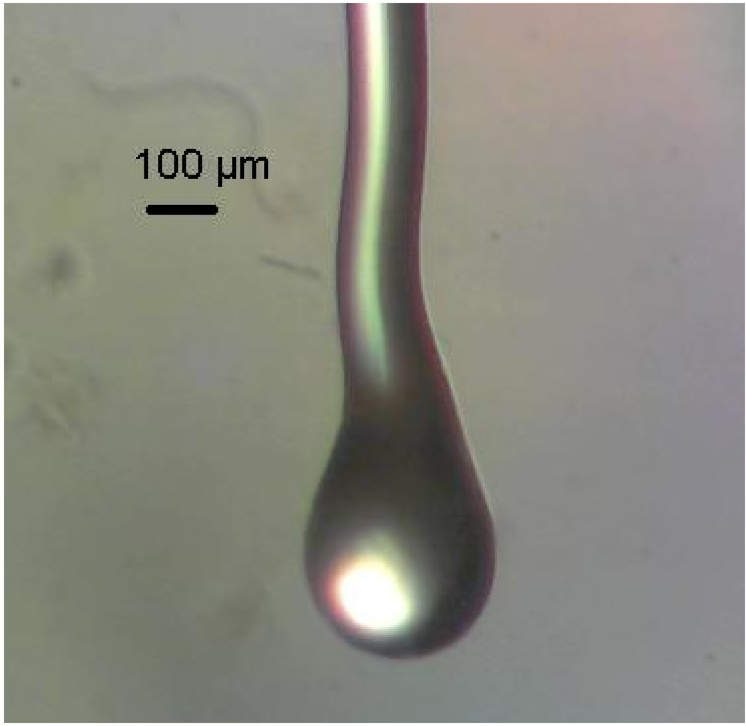
Pirms elektrolīzes pārtraukšanas, liesma jānodzēš ar slapju salveti vai nopūšot. Blakus jātur trauciņš ar ūdeni. Metāla adatas sprausla dažreiz aizdegas un nodeg. Tajā brīdī jānodzēš liesma ar slapju salveti.

Burbuļotājs sprādziena ierobežošanai



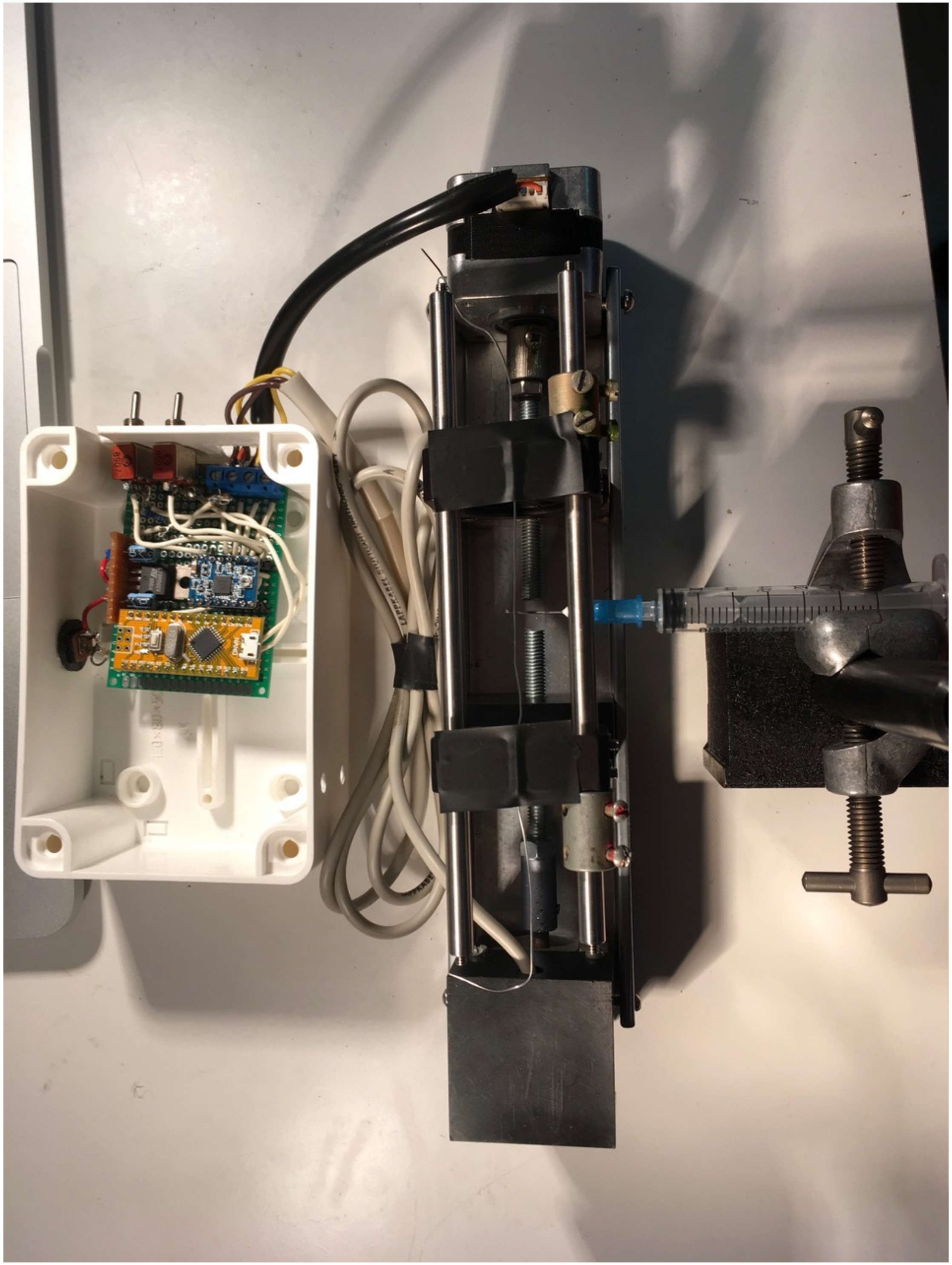
Rezonatora izgatavošanā šķiedras galā
(daudzmodu šķiedra)

Ievietojot liesmā no aizsargapvalka attīrītu optisko šķiedru, tā spoži iekvēlojas. Nepieciešamas saulesbrilles. Izgatavotais ČGMR resonators skatoties ar aci ir optiski caurspīdīgs.



Rezonators izgatavots šķiedras galā
(vienmodas šķiedra)

Kausējot vienmodas šķiedru, mēs saskārāmies ar komplikāciju - strauji plūstošā gāzes liesmas deformē un noliec plāno šķiedru. Šķiedra ir jārotē, lai mazinātu šo efektu.



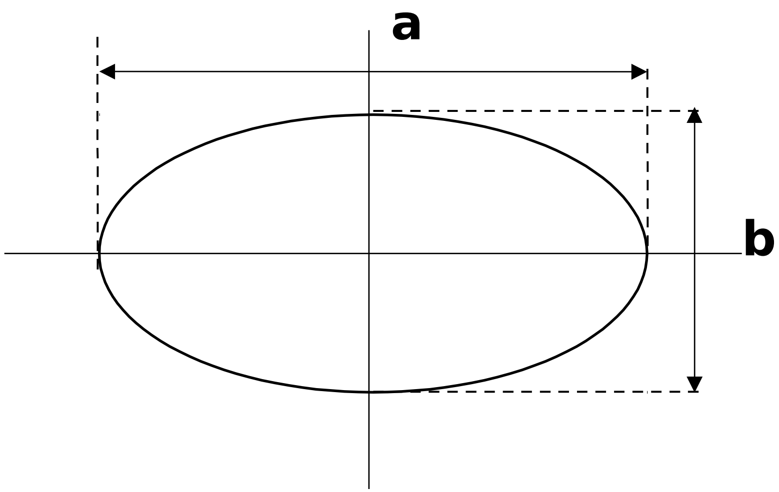
125µm šķiedru iespējams izstiept un pārdedzināt. Lai izgatavotu trapecveida šķiedru nepieciešams stiepšanas soļu motors, ar rokām nav iespējams tik vienmērīgi noturēt. Trapecveida šķiedra var saplīst pat no stiprāka elpas pūtiena.

Šķiedras stiepšanas soļu motors stiepj šķiedru uz abām pusēm, stiepšanas ātrumu ir iespējams regulēt, kamēr šķiedra tiek kausēta ar ūdeņraža liesmu. Vēl nepieciešams mikroskops, ar kuru tiks vērots un kontrolēts šķiedras stiepšanas process.

Rezonatoru deformācijas pakāpe

Degree of deforamation is calculated as follows[1]:

$$\eta = \frac{a-b}{a+b}$$



Degree of deformation was calculated for 2 optical fiber resonators: one melted on a single mode fiber, other on a multi mode fiber.

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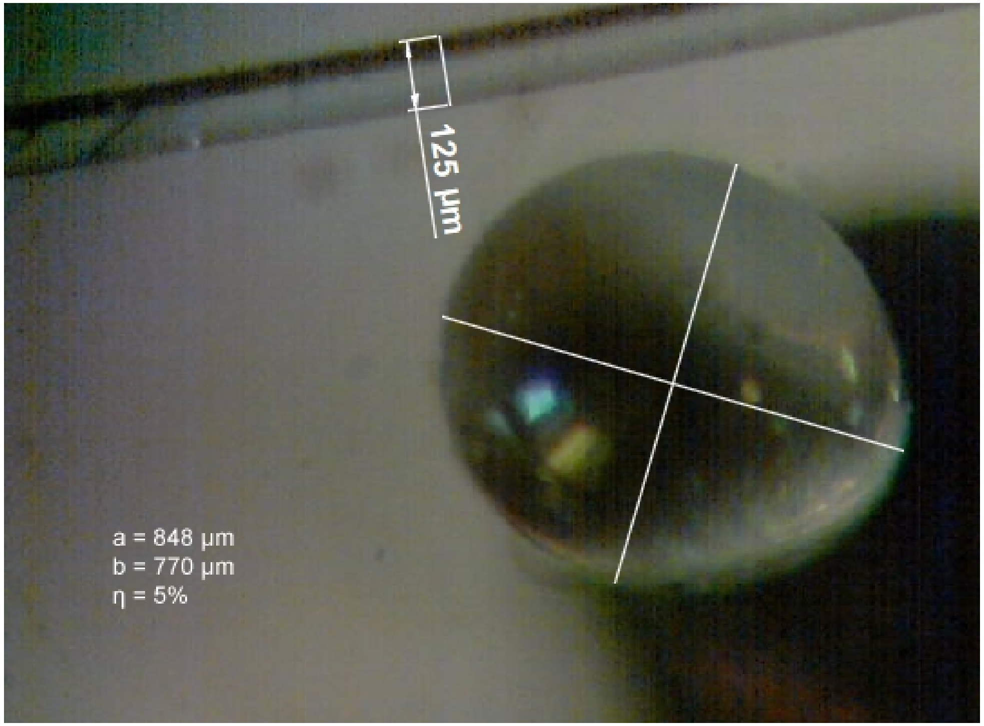
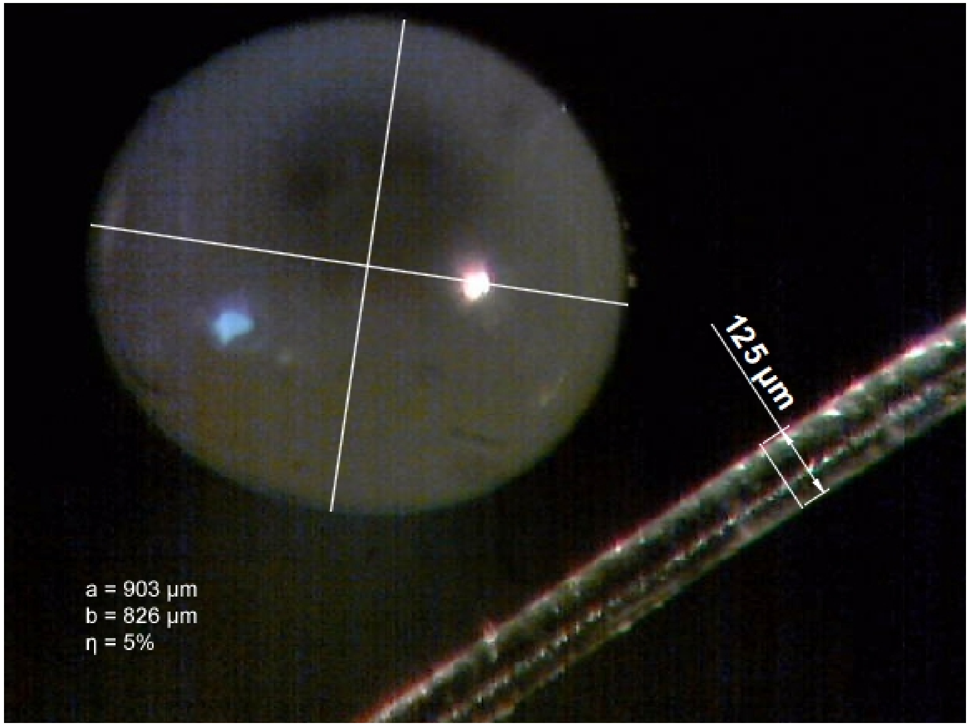
[1] Jun Tang, Jianhua Liu, Chenglong Shang, Chengfeng Xie, Hao Guo, Kun Qian, Chenyang Xue, Jun Liu, Fabrication and spectral characterizations of high asymmetric resonant cavities, Optics Communications, Volume 355, 2015, Pages 269-273, ISSN 0030-4018, http://dx.doi.org/10.1016/j.optcom.2015.06.077.

Deformācijas pakāpe tika aprēķināta diviem optisko šķiedru rezonatoriem: viens uzkausēts vienmodas šķiedras galā, bet otrs daudzmodu šķiedras galā.



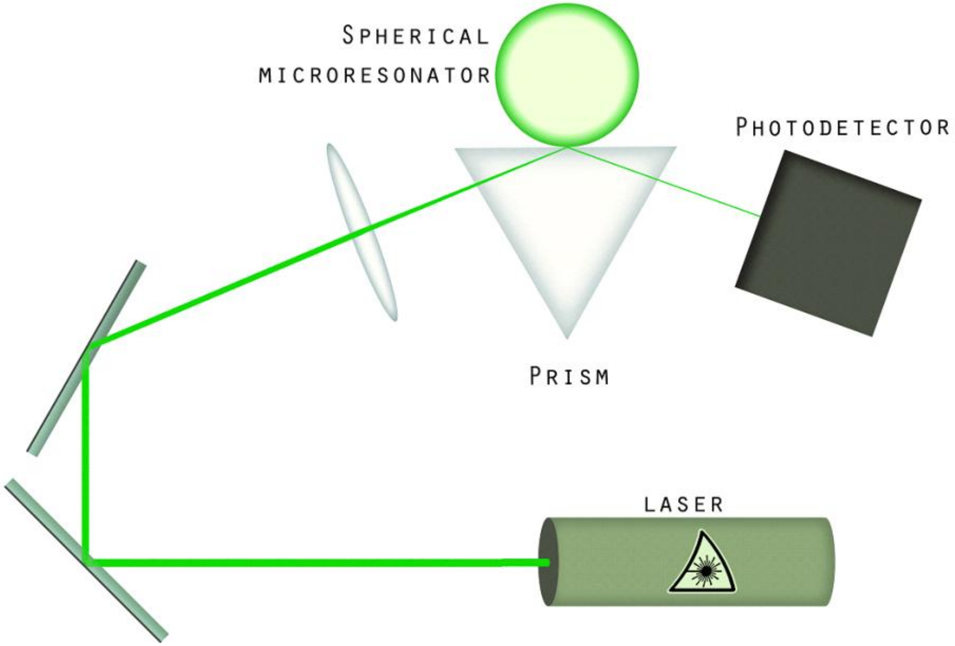
Vienmodas



Daudzmodu

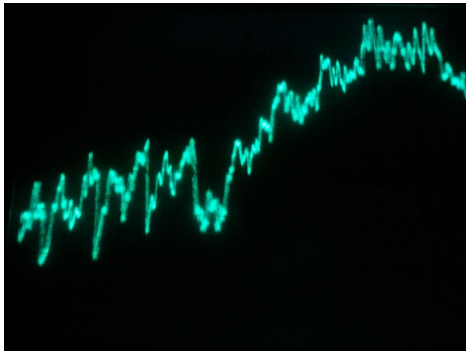
Rezonatora testēšana

Izgatavotie ČGMR tika ievietoti optiskajā sistēmā. Testēšanai tiek lietoti divi lāzeri. Zaļais lāzers 532 nm un infrasarkanais lāzers 760 nm.

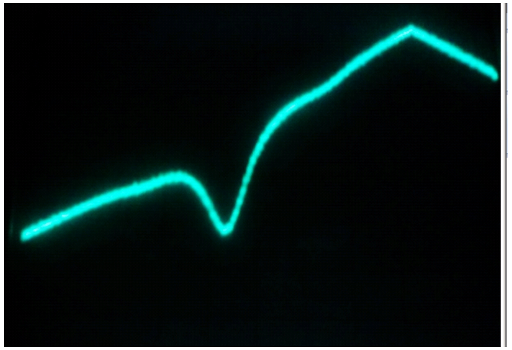


Optiskās sistēmas blokshēma

Mērot ar skenējama viļņa garuma VCSEL lāzeri 760 nm apgabalā, mainās redzamais fotodiodes signāls gadījumā, kad resonators piespiests prizmai un atvirzīts nost.



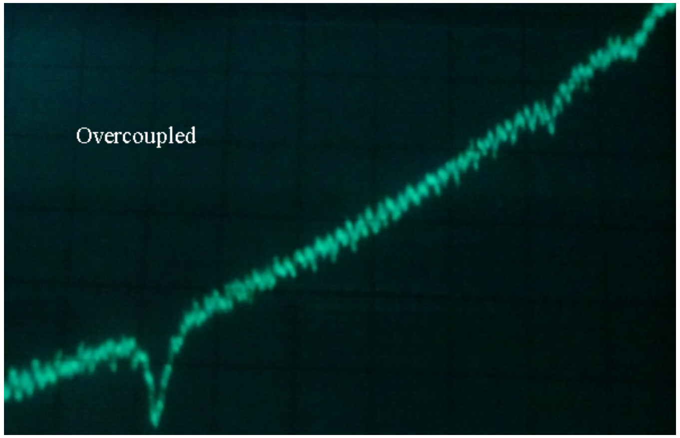
Piespiests



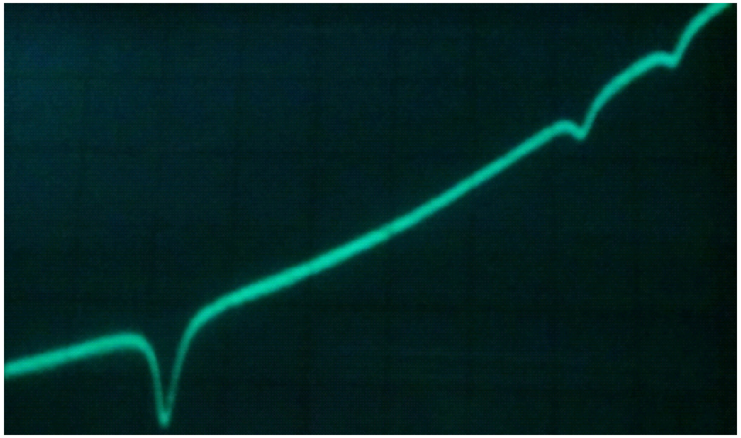
Atvirzīts

Rezonanšu platums atkarībā no attāluma līdz prizmai

Vēl ar mikrometru skrūvi tika mainīts ČGMR attālums līdz prizmai. Piespiežot pavisam klāt, rezonanses paliek platas, it kā izsmērējas un to dziļums samazinās. Savukārt pārāk lielā attālumā ČGM rezonanses pazūd, un ir redzamas tikai O_2 līnijas.

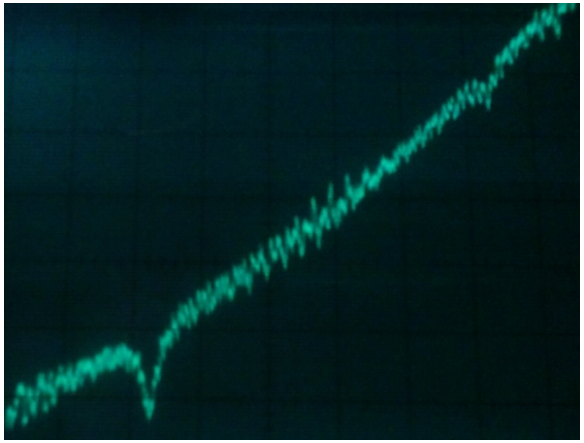


Overcoupled



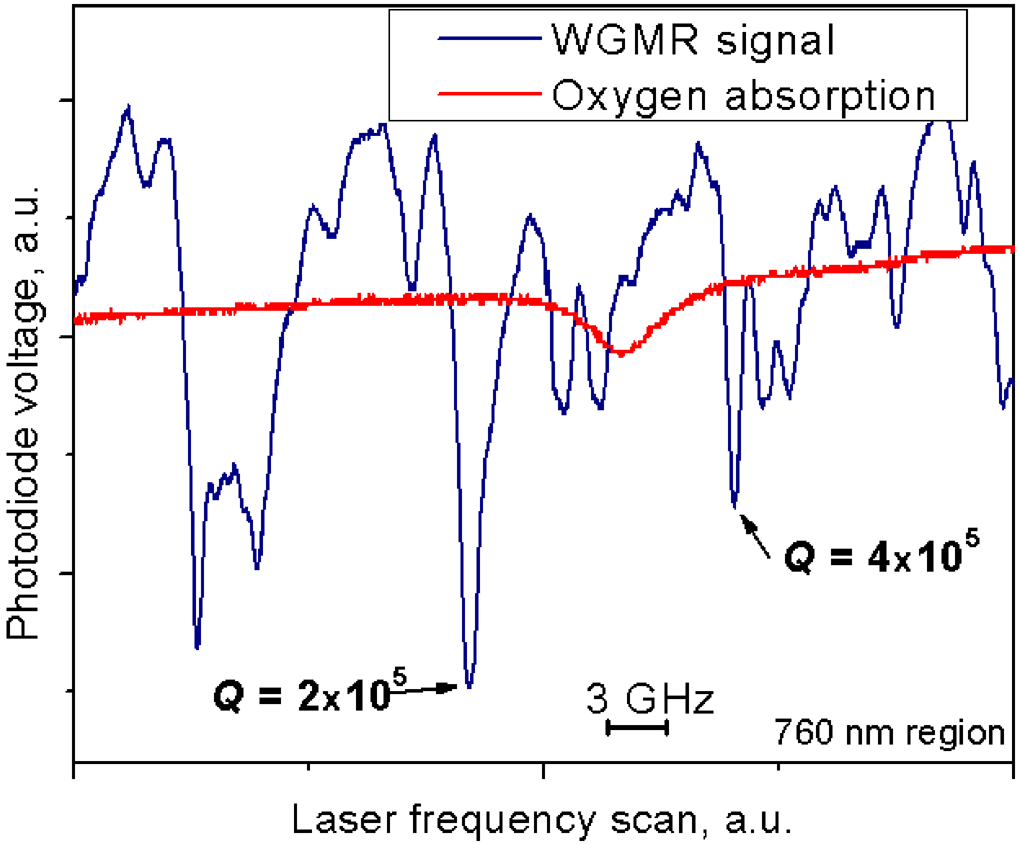
Undercoupled

Optimālajā attālumā rezonanses ir maksimāli šauras un dziļas.



Optimal

Rezonatora labuma faktora noteikšana



- Sfēriskos ČGM rezonatoros vainelaicīgi iespējams ierosināt dažādas azimutālās šķērsmodu sērijas. Ir sarežģīti atrast nosacījumus, lai šīs modu sērijas nepārklājas spektrā.

- Pielāgojot spoguļu novietojumu, mēs varējām noregulēt uz šaurās rezonanses un iegūt Q faktorus 10^5 diapazonā.

- Quality factor can be calculated as

  $$Q = \omega\tau,$$

  where τ is the photon lifetime and $\omega = 2\pi\nu$ is the optical frequency (ν is the scanning frequency).

  Photon lifetime is related to the FWHM linewidth as

  $$\tau = \frac{1}{\Delta\omega} = \frac{1}{2\pi\Delta\nu}.$$

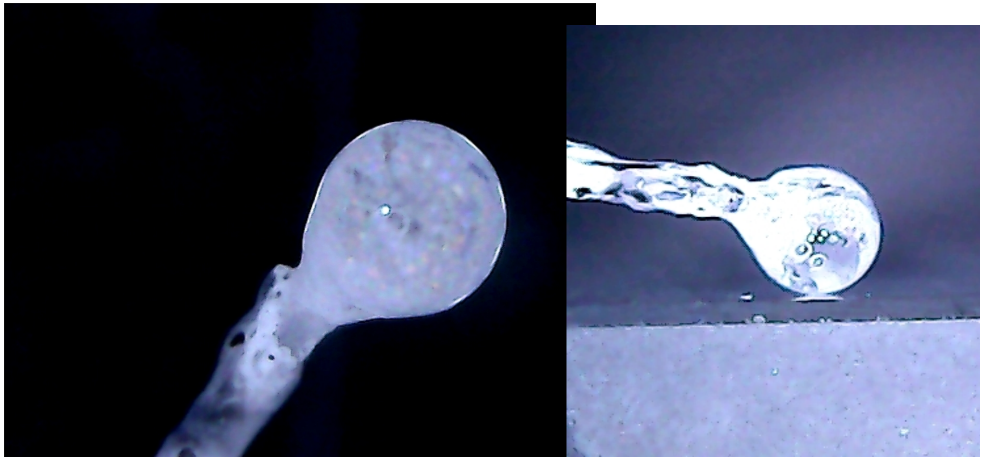
  In our experiments we had a resonator with $Q = 10^5$ and $\Delta\nu = 1\ \mathrm{GHz}$. So we calculated the photon lifetime and its traveled path :

  $$\tau = \frac{1}{10^9} = 1\ \mathrm{ns}$$

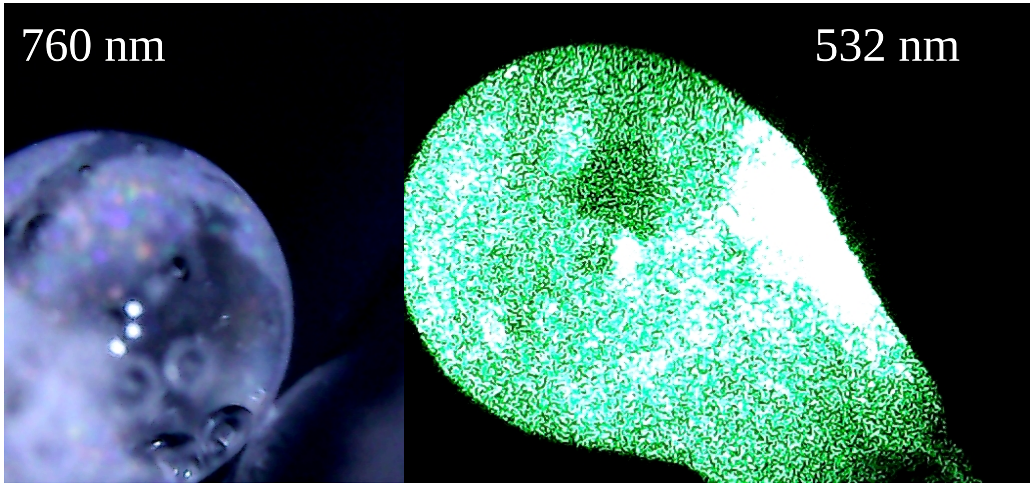
  $$L = c\tau = 3 \cdot 10^8 \cdot 10^{-9} = 0{,}3\ \mathrm{m}$$

  The short path explained why it was not possible to see the oxygen spectral lines.

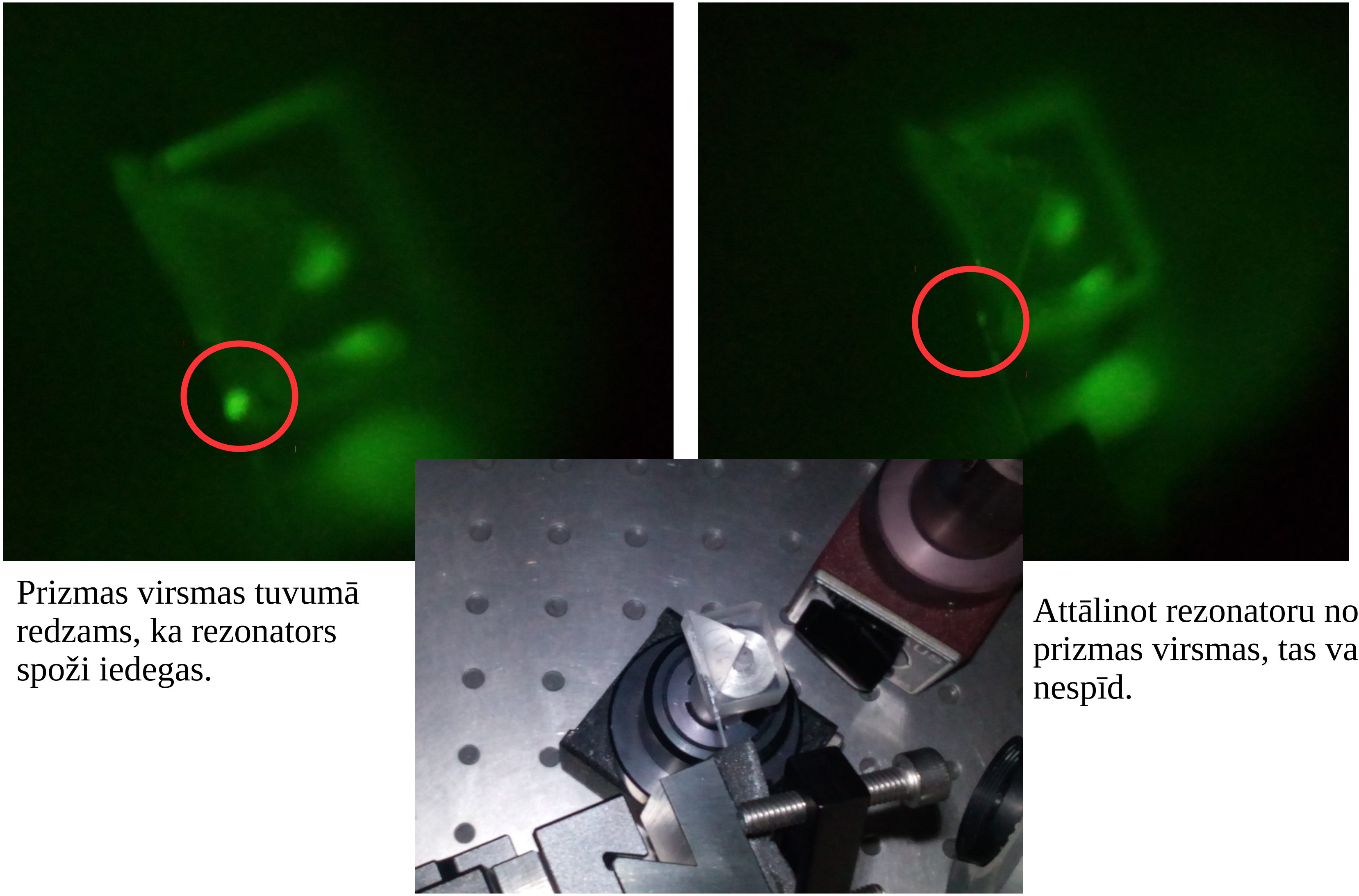
ČGM rezonanšu ierosināša rezonatorā, kas pārklāts ar polistirēna lodītēm.



- Šķiedras galā uzkausētais mikrorezonators ir pārklāts ar polistirēnan nanolodītēm. Vizuāli reonators nav pilnīgi caurspīdīgs, izskatās duļķains.



- Ievadot rezonatorā 532 nm lāzeri, redzams, ka rezonatora virsma samērā stipri izkliedē gaismu. Savukārt, ar 760 nm nepieciešama infrasarkanā kamera.

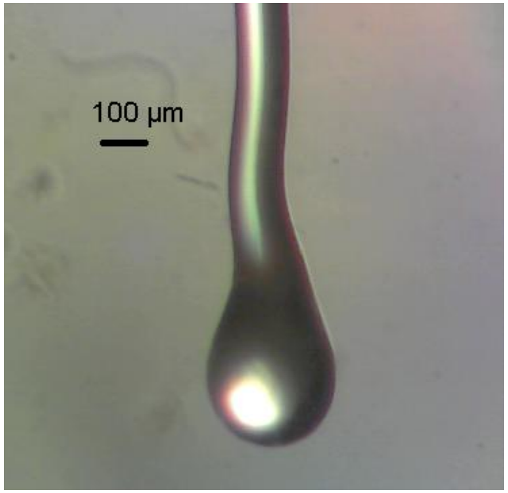


Prizmas virsmas tuvumā redzams, ka rezonators spoži iedegas.

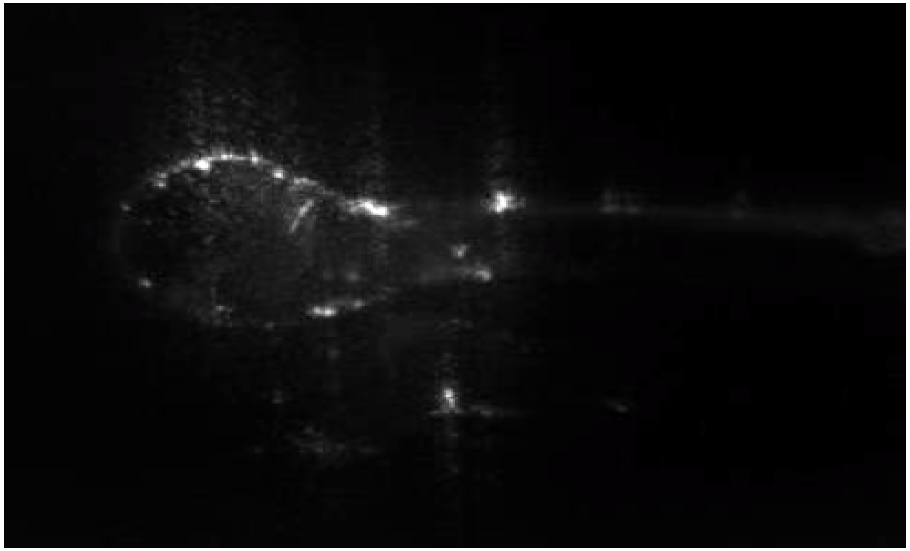
Attālinot rezonatoru no prizmas virsmas, tas vairs nespīd.

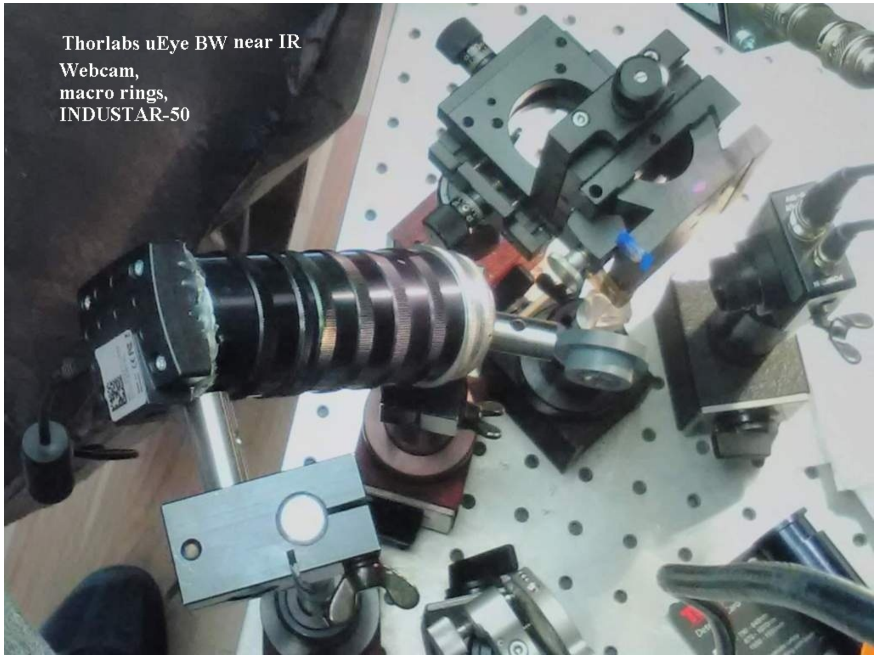
Attēli uzņemti izmantojot divas kameras (infrasarkanā kamera + fotoaparāts), tādēļ bildēm ir neass fokuss. Lai varētu uzņemt kvalitatīvākas bildes, meklējām citu kameru, ar kuru var vienlaicīgi gan redzēt, gan uzņemt bildes.

Melnbalta kamera rezonanšu novērošanai

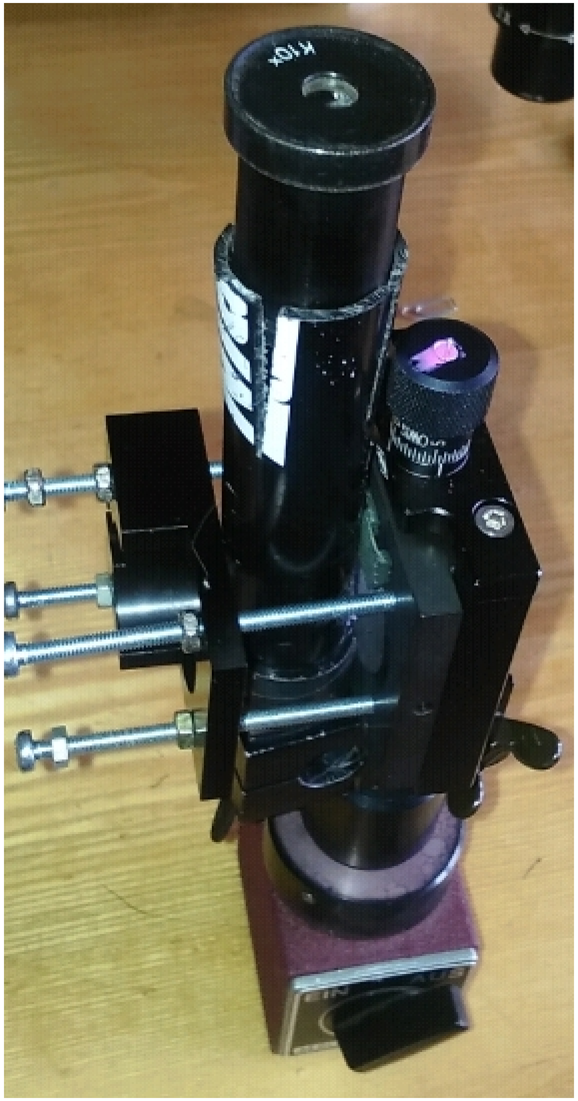


- Rezonanšu novērošanai testēšanas sistēma jāpapildina ar kameru, kas “redz” infrasarkano starojumu.
- Ar melnbalto ThorLabs kameru izdevās novērot pat atsevišķas rezonanses. Diemžēl kamera kopā ar objektīvu ir masīva Tiek veidota kompaktāka sistēma.

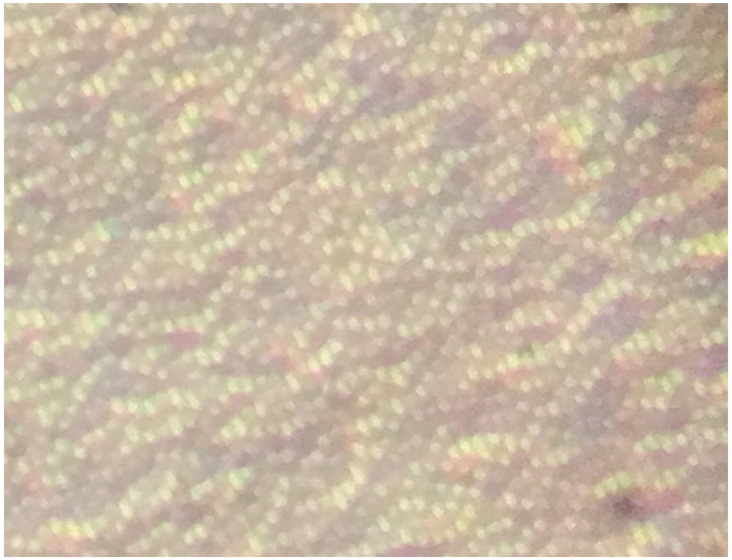




Mūsu pašizgatavotais mikroskops ar CD lasītāja lēcu



- Lai izgatavojat trapecveida šķiedru būtu iespējams sekot līdzi procesam un redzēt šķiedras diametru, nepieciešams mikroskops.
- Mēs izgatavojām mikroskopu, izmantojot CD lasītāja lēcu, objektīvu, cauruli, ar mikrometra skrūvi bīdāmu galdiņu.
- Ar šo mikroskopu bija iespējams saskatīt diska celiņus



Jaunu čukstošās galerijas modu mikrorezonatoru izstrāde optisko frekvenču standartu un biosensoru pielietojumiem, un to raksturošana ar femtosekunžu optisko frekvenču ķemmi

Otrā ceturkšņa atskaite

3. Modelēšana

NACIONĀLAIS ATTĪSTĪBAS PLĀNS 2020

EIROPAS SAVIENĪBA
Eiropas Reģionālās attīstības fonds

IEGULDĪJUMS TAVĀ NĀKOTNĒ

ERAF projekts Nr. 1.1.1.1/16/A/259

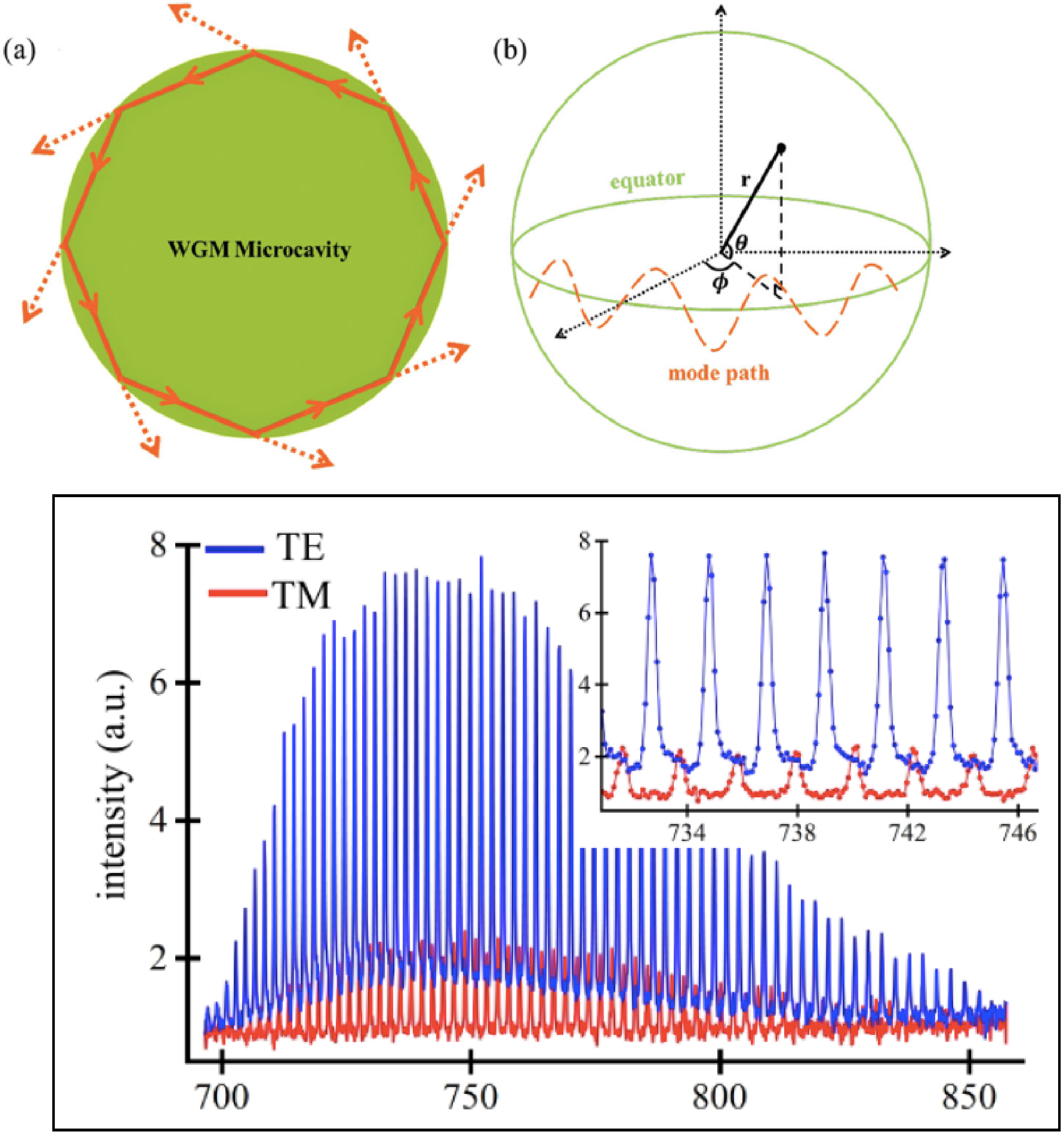


Figure 1.2: Spectra of Whisperring gallery mode sphere https://sites.ualberta.ca/~ameldrum/science/science4a.html

$$\boxed{2\pi r n = \lambda N}$$

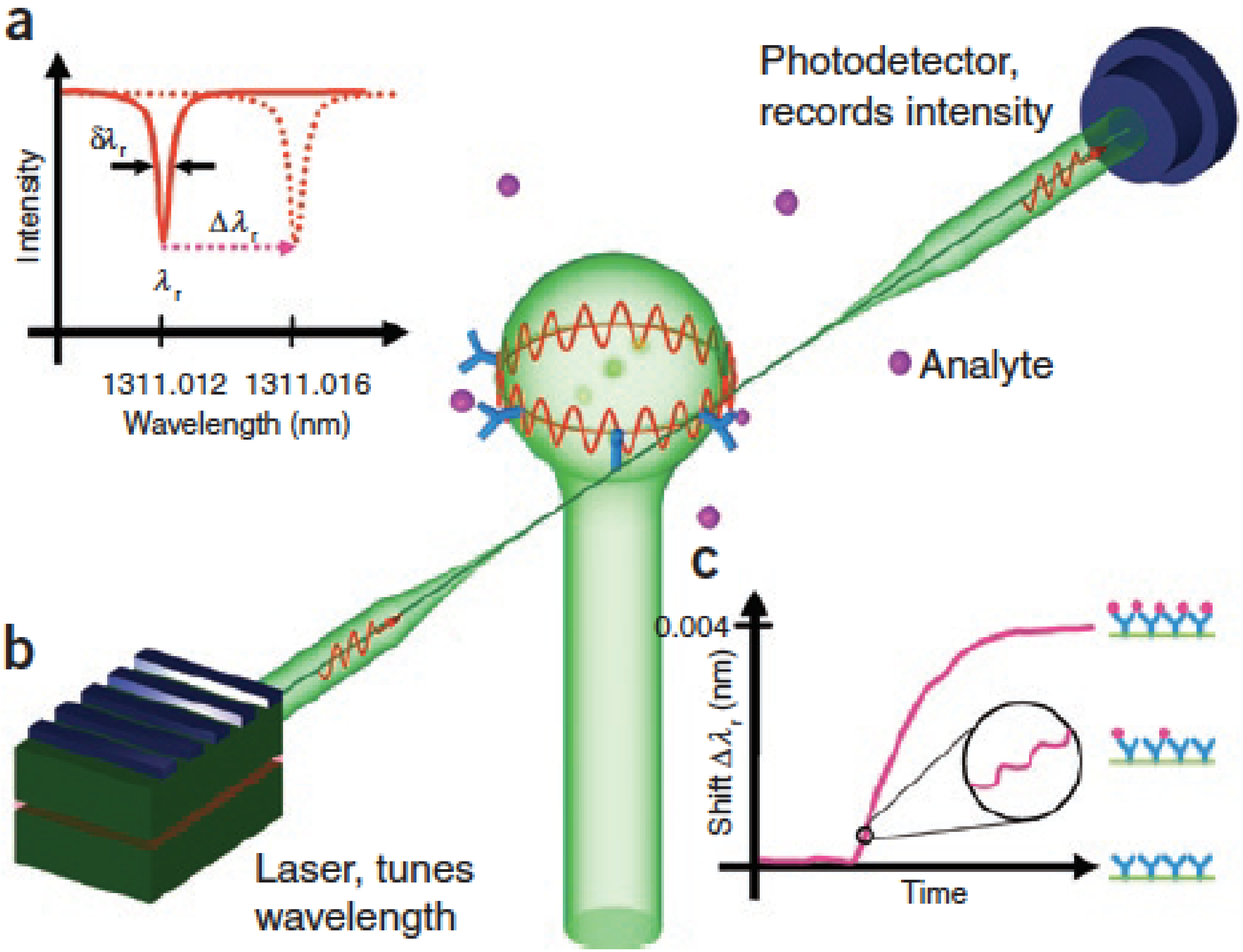
$$2\pi r n = (\lambda + \Delta\lambda)(N+1)$$

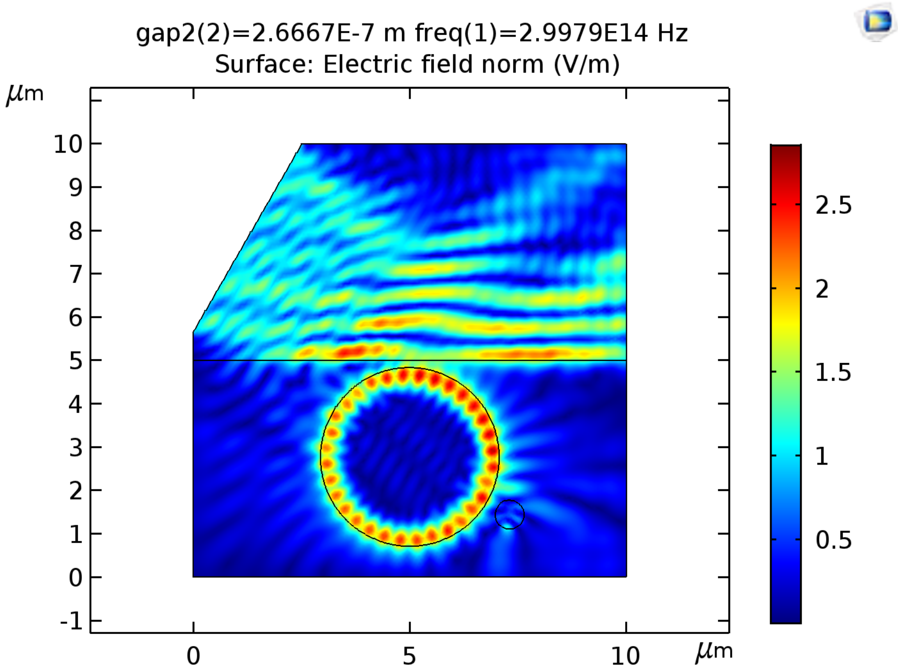
$$\boxed{\Delta\lambda \approx -\frac{\lambda^2}{2\pi r n}}$$

$$\boxed{\Delta\nu = \frac{c}{2\pi r n}}$$

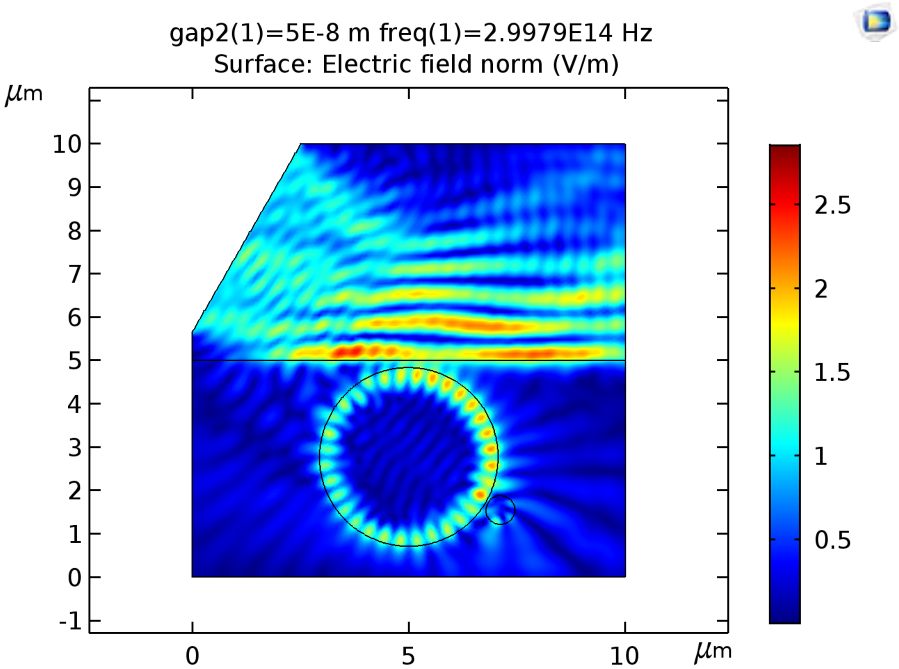
$$\lambda_{TE} \approx \frac{2\pi R n_1}{m + 1.856 m^{\frac{1}{3}} + \left(\frac{1}{2} - \frac{n_1}{\sqrt{n_1^2 - 1}}\right)}$$

Čukstošās galerijas modu mikrorezonators

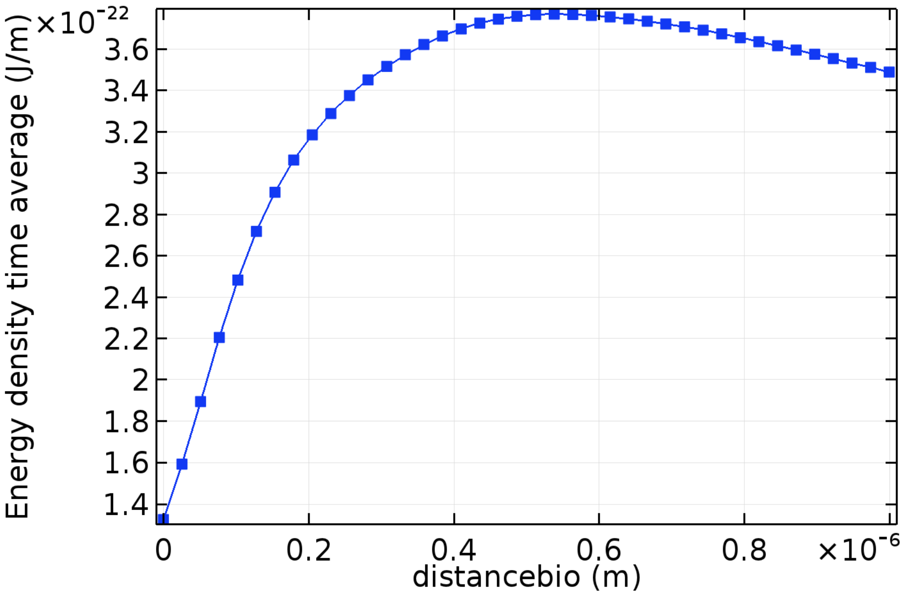




Daļiņa ir tālu no rezonatora



Daļiņa ir tuvu rezonatoram



Kad rezonatoram pietuvojas kāda daļiņa, tā var izmainīt rezonatora rezonanses frekvenci.

Ja daļiņa ir pietiekami tālu, tā nemijiedarbojas ar rezonatora virsmas vilni. Daļiņai tuvojoties mijiedarbība pastiprinās.

Ir optimālais attālums, kad mijiedarbība ir visstiprākā.

Maksvela vienādojumu rēķināšana un risinājuma attēlošana programmā Mathematica, sfēra

$$\begin{aligned}\nabla\times\vec{E}+\frac{\partial\vec{B}}{\partial t} &= 0\\ \nabla\times\vec{H}-\frac{\partial\vec{D}}{\partial t} &= \vec{J}\\ \nabla\cdot\vec{D} &= \rho\\ \nabla\cdot\vec{B} &= 0\end{aligned}$$

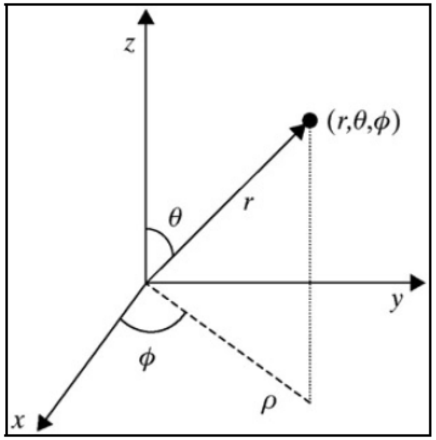
$$\begin{aligned}\vec{D} &= \varepsilon\vec{E}\\ \vec{H} &= \frac{1}{\mu}\vec{B}\\ \varepsilon &= \varepsilon_r\varepsilon_0\\ \mu &= \mu_r\mu_0\\ c^2 &= \frac{1}{\varepsilon_0\mu_0}\end{aligned}$$

$\rho = 0$ and $\vec{J} = 0.$

$$\Delta\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\vec{E}(\vec{r},t)=\vec{E}(\vec{r})*e^{-i\omega t} \qquad k^2=\mu\varepsilon\omega^2.$$

$$\boxed{\Delta\vec{E}+k^2\vec{E}=0}$$



$$\Delta=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$$\begin{aligned}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\vec{E}(\vec{r})}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\vec{E}(\vec{r})}{\partial\theta}\right)&\\ +\frac{1}{r^2\sin^2\theta}\frac{\partial^2\vec{E}(\vec{r})}{\partial\phi^2}+k^2\vec{E}(\vec{r}) &= 0\end{aligned}$$

$$\vec{E}(\vec{r}) = E_r(r,\phi,\theta)\vec{i}_r + E_\phi(r,\phi,\theta)\vec{i}_\phi + E_\theta(r,\phi,\theta)\vec{i}_\theta$$

$$\vec{E}(\vec{r}) = E_\theta(r,\phi,\theta)\vec{i}_\theta$$

$$E = E_\theta = \Psi_r(r)\cdot\Psi_\theta(\theta)\cdot\Psi_\phi(\phi)$$

$$\begin{aligned} \frac{1}{\Psi_r}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_r}{\partial r}\right) + k^2r^2 + \\ \frac{1}{\sin^2\theta}\left[\frac{\sin\theta}{\Psi_\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi_\theta}{\partial\theta}\right) + \frac{1}{\Psi_\phi}\frac{\partial^2\Psi_\phi}{\partial\phi^2}\right] &= 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{\Psi_r}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi_r}{\partial r}\right) + k^2r^2 &= C_1 \\ \frac{1}{\Psi_\phi}\frac{\partial^2\Psi_\phi}{\partial\phi^2} &= C_2 \end{aligned}$$

$$C_1 + \frac{1}{\sin^2\theta}\left[\frac{\sin\theta}{\Psi_\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi_\theta}{\partial\theta}\right) + C_2\right] = 0$$

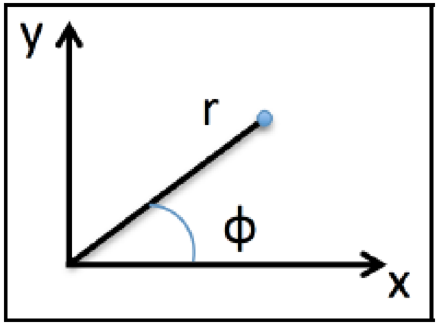
Derivation of Ψ_ϕ

$$\frac{1}{\Psi_\phi}\frac{\partial^2\Psi_\phi}{\partial\phi^2} = C_2$$

$$\Psi_\phi = \Psi_{\phi+2\pi}.$$

$$\frac{d^2\Psi_\phi}{d\phi^2} + m^2\Psi_\phi = 0$$

$$m = 0, \pm1, \pm2, ...$$



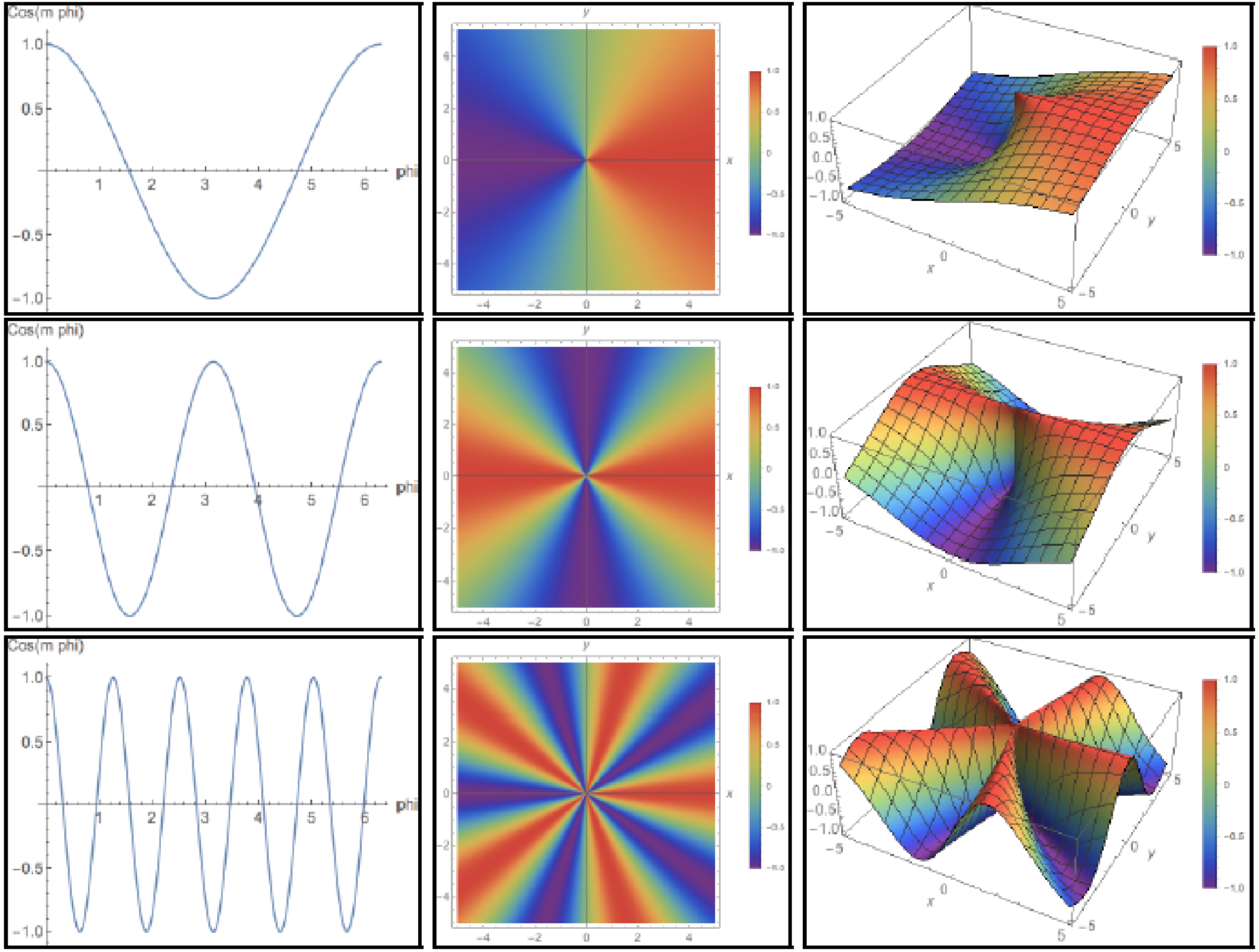
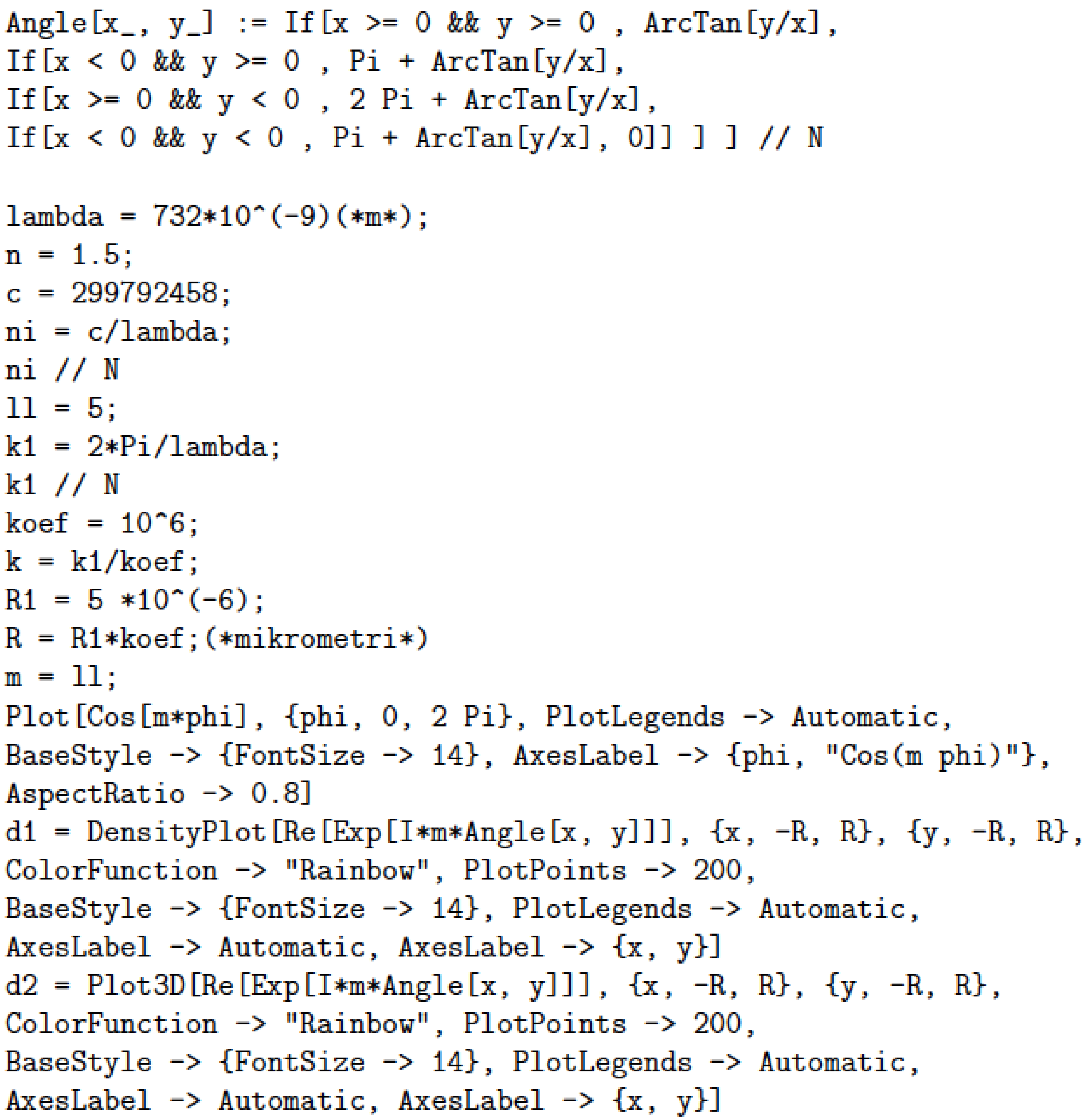


Figure 7.3: Simulation in Wolfram Mathematica. The value of $Re\left(e^{i\cdot m\phi}\right) = \cos(m\phi)$ when $m = 1$, 2 and 5.



```
Angle[x_, y_] := If[x >= 0 && y >= 0 , ArcTan[y/x],
If[x < 0 && y >= 0 , Pi + ArcTan[y/x],
If[x >= 0 && y < 0 , 2 Pi + ArcTan[y/x],
If[x < 0 && y < 0 , Pi + ArcTan[y/x], 0]] ] ] // N

lambda = 732*10^(-9)(*m*);
n = 1.5;
c = 299792458;
ni = c/lambda;
ni // N
ll = 5;
k1 = 2*Pi/lambda;
k1 // N
koef = 10^6;
k = k1/koef;
R1 = 5 *10^(-6);
R = R1*koef;(*mikrometri*)
m = ll;
Plot[Cos[m*phi], {phi, 0, 2 Pi}, PlotLegends -> Automatic,
BaseStyle -> {FontSize -> 14}, AxesLabel -> {phi, "Cos(m phi)"},
AspectRatio -> 0.8]
d1 = DensityPlot[Re[Exp[I*m*Angle[x, y]]], {x, -R, R}, {y, -R, R},
ColorFunction -> "Rainbow", PlotPoints -> 200,
BaseStyle -> {FontSize -> 14}, PlotLegends -> Automatic,
AxesLabel -> Automatic, AxesLabel -> {x, y}]
d2 = Plot3D[Re[Exp[I*m*Angle[x, y]]], {x, -R, R}, {y, -R, R},
ColorFunction -> "Rainbow", PlotPoints -> 200,
BaseStyle -> {FontSize -> 14}, PlotLegends -> Automatic,
AxesLabel -> Automatic, AxesLabel -> {x, y}]
```

Kods
programmā
Mathematica

Derivation of Ψ_θ

$$C_1 + \frac{1}{\sin^2\theta}\left[\frac{\sin\theta}{\Psi_\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi_\theta}{\partial\theta}\right) - m^2\right] = 0 \qquad \eta = \cos\theta$$

$$(1-\eta^2)\frac{d^2\Psi_\eta}{d\eta^2} - 2\eta\frac{d\Psi_\eta}{d\eta} + \left(C_1 - \frac{m^2}{1-\eta^2}\right)\Psi_\eta = 0$$

This equation is similar to Associated Legendre Equation ([1], equation 8.1.1.) and ([96], equation 14.2.2.) :

$$(1-z^2)\frac{d^2\omega}{dz^2} - 2z\frac{d\omega}{dz} + \left(\nu(\nu+1) - \frac{\mu^2}{1-z^2}\right)\omega = 0 \tag{7.153}$$

$$C_1 = l(l+1)$$

$$(1-\eta^2)\frac{d^2\Psi_\eta}{d\eta^2} - 2\eta\frac{d\Psi_\eta}{d\eta} + \left(l(l+1) - \frac{m^2}{1-\eta^2}\right)\Psi_\eta = 0$$

Standard solutions are ([96], section 14.2.2.):

- Ferrers functions of the first kind (also called Legendre Functions on the cut) $\mathrm{P}_l^m(\pm\eta)$ and $\mathrm{P}_l^{-m}(\pm\eta)$
- Ferrers functions of the second kind (also called Legendre Functions on the cut) $\mathrm{Q}_l^m(\pm\eta)$ and $\mathrm{Q}_{-l-1}^m(\pm\eta)$
- Associated Legendre Functions of first kind $P_l^m(\pm\eta)$ and $P_l^{-m}(\pm\eta)$
- Associated Legendre Functions of second kind $\boldsymbol{Q}_l^m(\pm\eta)$ and $\boldsymbol{Q}_{-l-1}^m(\pm\eta)$

$$\mathrm{P}_l^m(\eta) = \left(\frac{1+\eta}{1-\eta}\right)^{m/2} \mathbf{F}\left(l+1, -l; 1-m; \frac{1-\eta}{2}\right) \qquad \mathbf{F}\,(a,b;c;x) = \frac{1}{\Gamma(c)} F(a,b;c;x)$$

[96] Frank W. J. Olver, Daniel W. Lozier, Ronal F. Biosvert, and Charles W. Clark, editors. *NIST Handbook of Mathematical Functions*. National Institute of Standards and Technology U.S. Department of Commerce and Cambridge University Press, 2010.

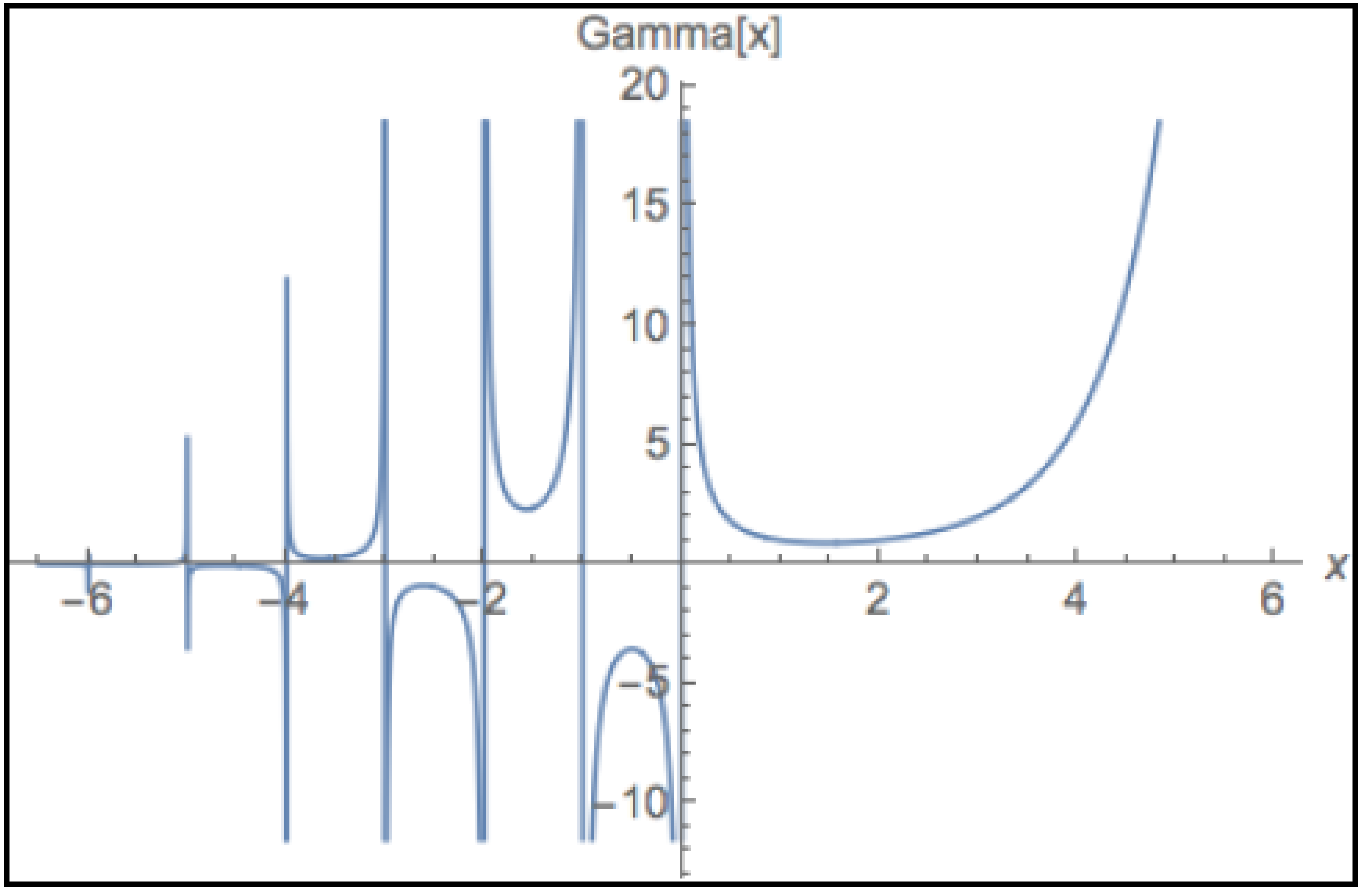


Figure 7.4: Simulation in Wolfram Mathematica. Gamma[x]

If m=0,1,2,.. the equation is reduced to ([96], equation 14.3.4)

$$\mathrm{P}_l^m(\eta) = (-1)^m \frac{\Gamma(l+m+1)}{2^m \Gamma(l-m+1)} \left(1-\eta^2\right)^{m/2} \mathbf{F}\left(l+m+1, m-l; m+1; \frac{1-\eta}{2}\right) \tag{7.166}$$

$$\Psi_\theta = \Psi_\eta = A_{lm} P_l^{|m|}(cos\theta)$$

Table 7.2: Qualitative analyses of $P_n^m(\cos\theta)$

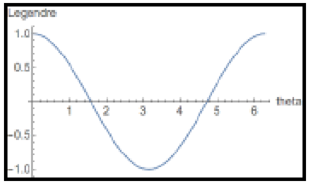
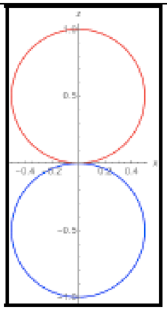
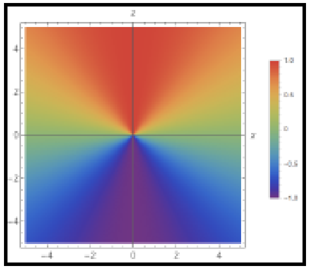
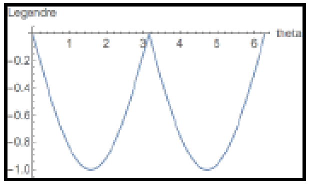
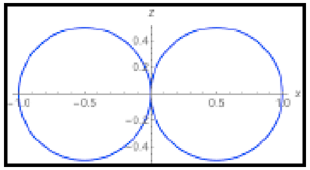
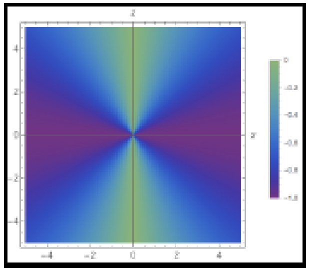
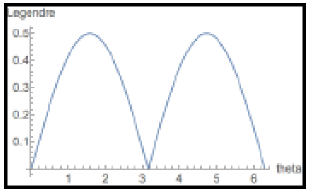
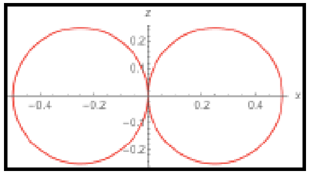
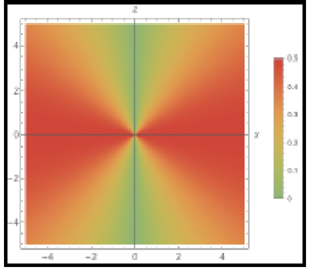
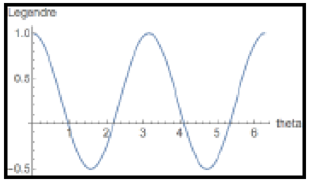
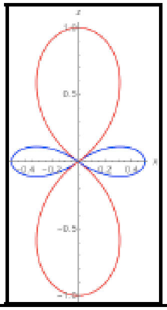
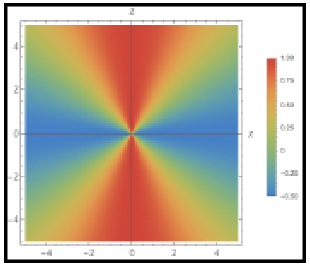
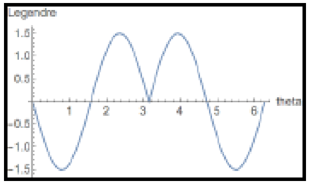
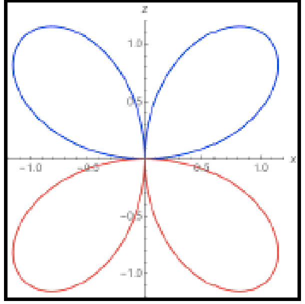
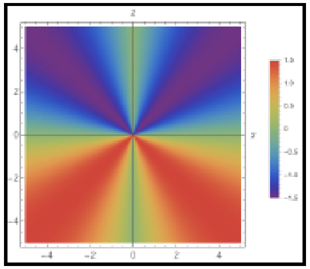
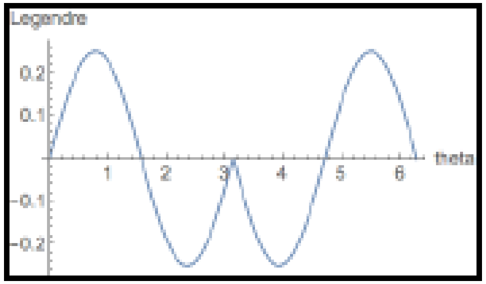
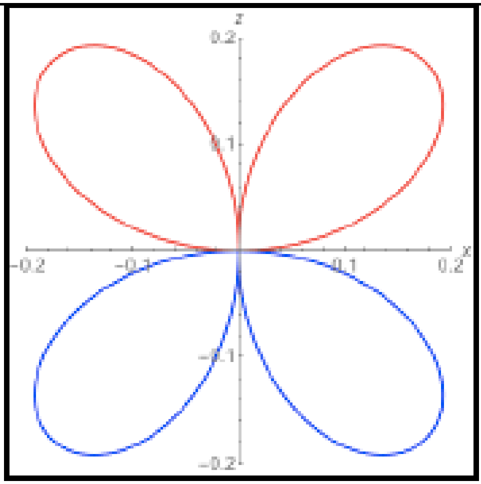
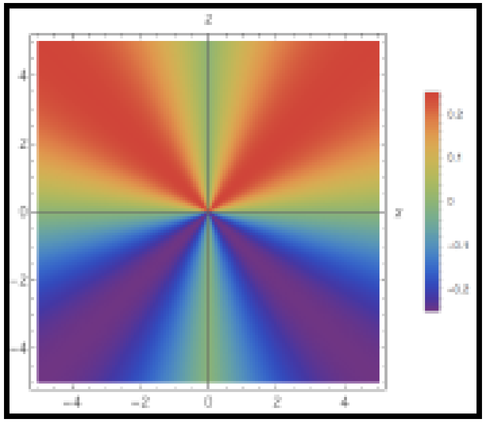
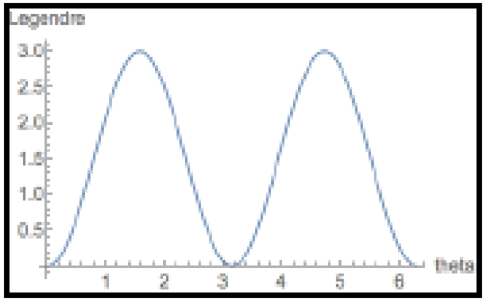
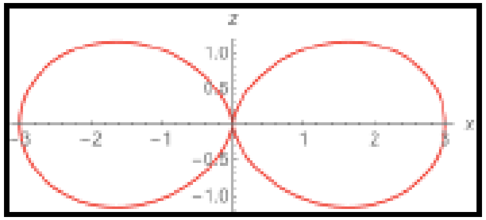
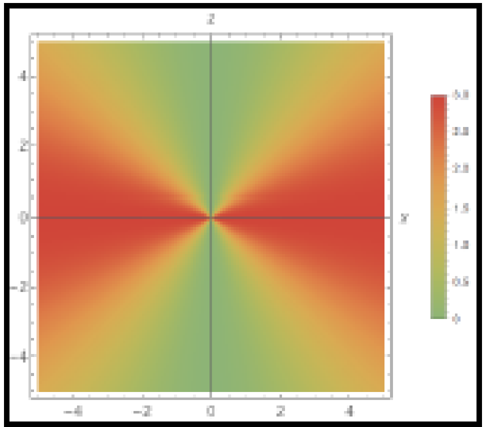
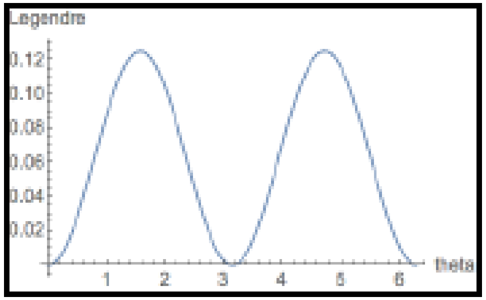
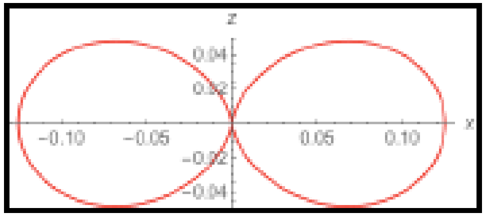
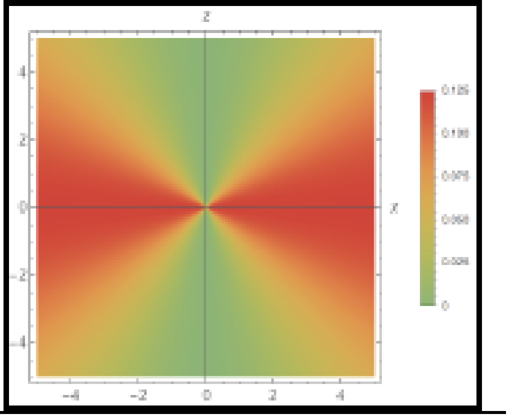
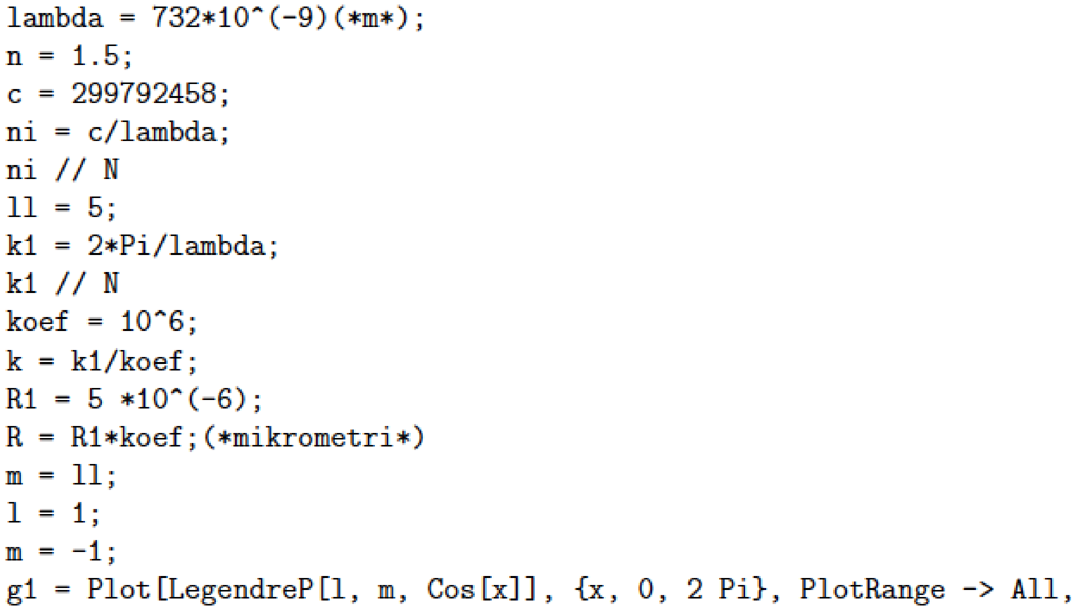
Parameter values	Plot	PolarPlot	DensityPlot
$l = 1\ m = 0$			
$l = 1\ m = 1$			
$l = 1\ m =$ -1			
$l = 2\ m = 0$			
$l = 2\ m = 1$			

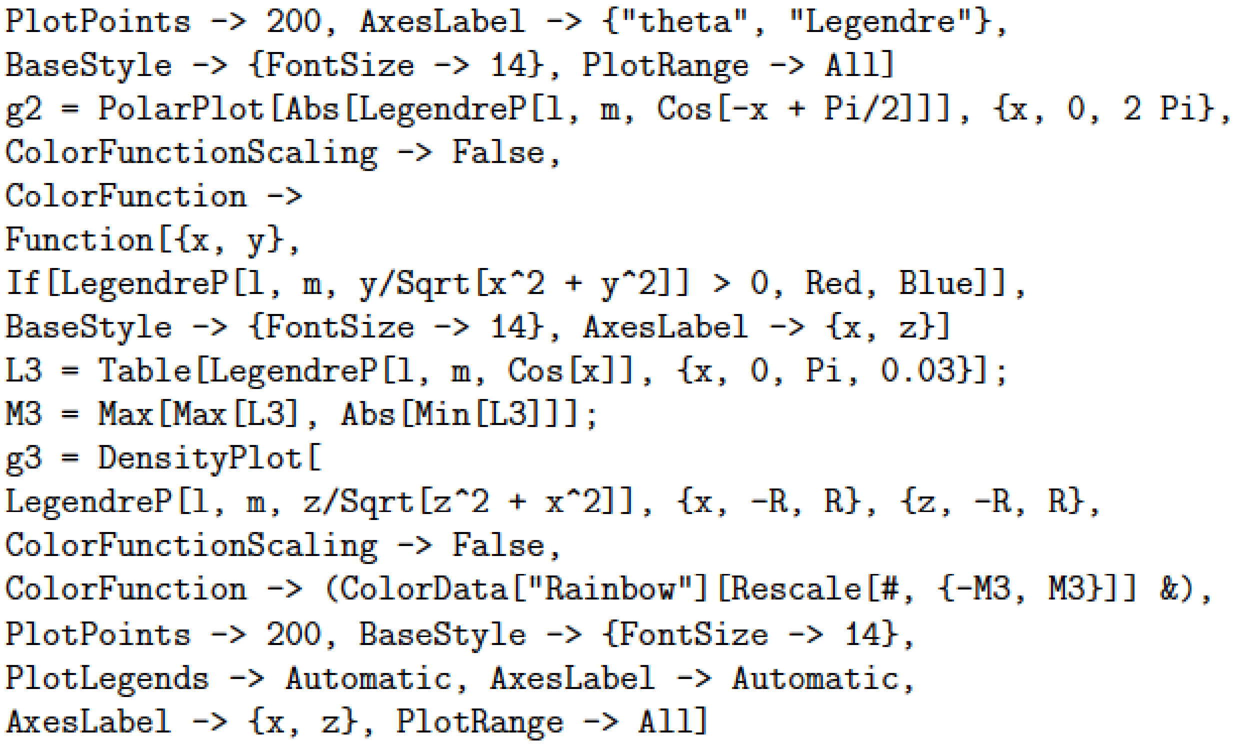
Table continued . . .

Parameter values	Plot	PolarPlot	DensityPlot
$l = 2\ m =$ -1			
$l = 2\ m = 2$			
$l = 2\ m =$ -2			

Kods programmā Mathematica



```
lambda = 732*10^(-9)(*m*);
n = 1.5;
c = 299792458;
ni = c/lambda;
ni // N
ll = 5;
k1 = 2*Pi/lambda;
k1 // N
koef = 10^6;
k = k1/koef;
R1 = 5 *10^(-6);
R = R1*koef;(*mikrometri*)
m = ll;
l = 1;
m = -1;
g1 = Plot[LegendreP[l, m, Cos[x]], {x, 0, 2 Pi}, PlotRange -> All,
```



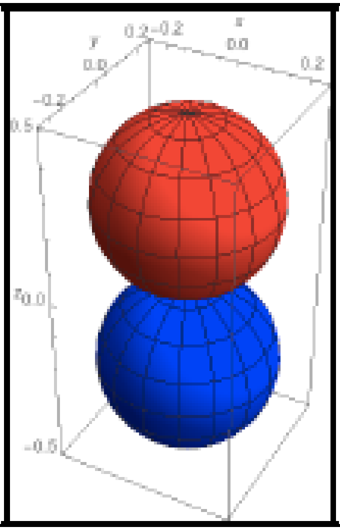
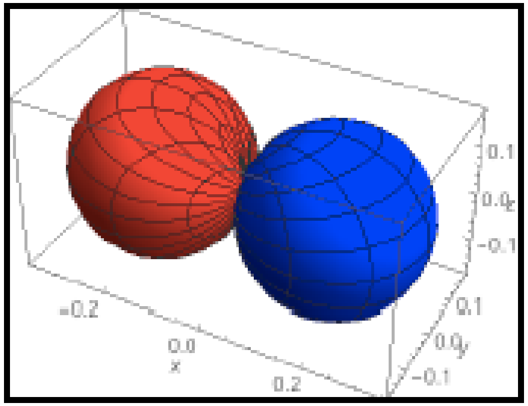
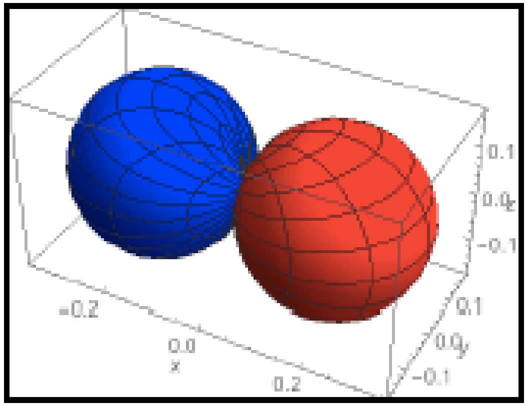
```
PlotPoints -> 200, AxesLabel -> {"theta", "Legendre"},
BaseStyle -> {FontSize -> 14}, PlotRange -> All]
g2 = PolarPlot[Abs[LegendreP[l, m, Cos[-x + Pi/2]]], {x, 0, 2 Pi},
ColorFunctionScaling -> False,
ColorFunction ->
Function[{x, y},
If[LegendreP[l, m, y/Sqrt[x^2 + y^2]] > 0, Red, Blue]],
BaseStyle -> {FontSize -> 14}, AxesLabel -> {x, z}]
L3 = Table[LegendreP[l, m, Cos[x]], {x, 0, Pi, 0.03}];
M3 = Max[Max[L3], Abs[Min[L3]]];
g3 = DensityPlot[
LegendreP[l, m, z/Sqrt[z^2 + x^2]], {x, -R, R}, {z, -R, R},
ColorFunctionScaling -> False,
ColorFunction -> (ColorData["Rainbow"][Rescale[#, {-M3, M3}]] &),
PlotPoints -> 200, BaseStyle -> {FontSize -> 14},
PlotLegends -> Automatic, AxesLabel -> Automatic,
AxesLabel -> {x, z}, PlotRange -> All]
```

Analyses of $Y_l^m(\theta, \phi) = \Psi_\theta \Psi_\phi$

Wolfram Mathematica has a function `SphericalHarmonicY[`l,m, θ, ϕ`]`[33] with the folloowing value

$$Y_l^m(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l-m)!}{(l+m)!}} P_l^m(cos\theta) e^{im\phi} \tag{7.177}$$

Table 7.3: Qualitative analyses of $P_n^m(\cos\theta)$

	m = 0	m=1	m=-1
$l = 1$			

Derivation of Ψ_r

$$\frac{1}{\Psi_r}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Psi_r}{\partial r}\right) + k^2r^2 \;=\; l(l+1) \qquad\qquad u = kr.$$

$$u^2\frac{\partial^2 \Psi_u}{\partial u^2} + 2u\frac{\partial \Psi_u}{\partial u} + (u^2 - l(l+1))\Psi_u \;=\; 0$$

This equation is similar to equation of Spherical Bessel Functions ([1], equation 10.1.1; [96], equation 10.47.1.)

$$u^2\frac{\partial^2 \omega}{\partial u^2} + 2u\frac{\partial \omega}{\partial u} + (u^2 - l(l+1))\omega \;=\; 0 \tag{7.190}$$

Spheric Bessel Function of first kind ([1], equation 10.1.2; [96], equation 10.47.3.):

$$j_l(u) = \sqrt{\frac{\pi}{2u}}J_{l+1/2}(u) \tag{7.194}$$

where $J_l(u)$ is Bessel Function of the First kind ([96], equation 10.2.2.)

$$J_l(u) = \left(\frac{1}{2}u\right)^n \sum_{k=0}^{\infty}(-1)^k\frac{\left(\frac{1}{4}u^2\right)^k}{k!\Gamma(l+k+1)} \tag{7.195}$$

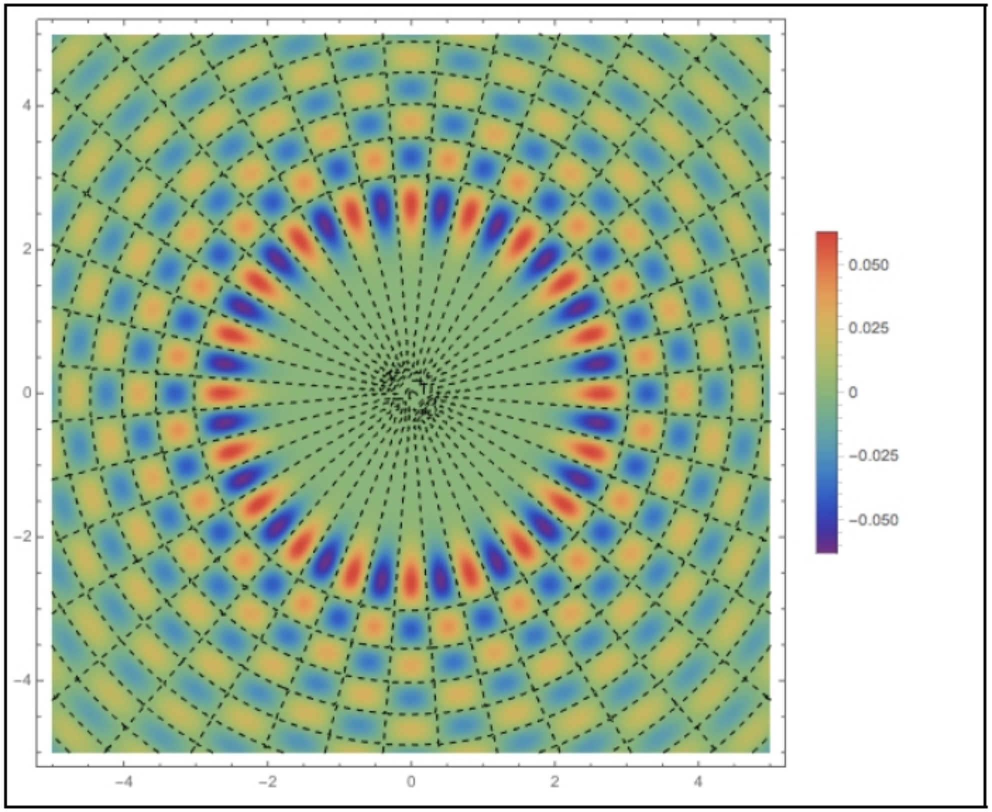
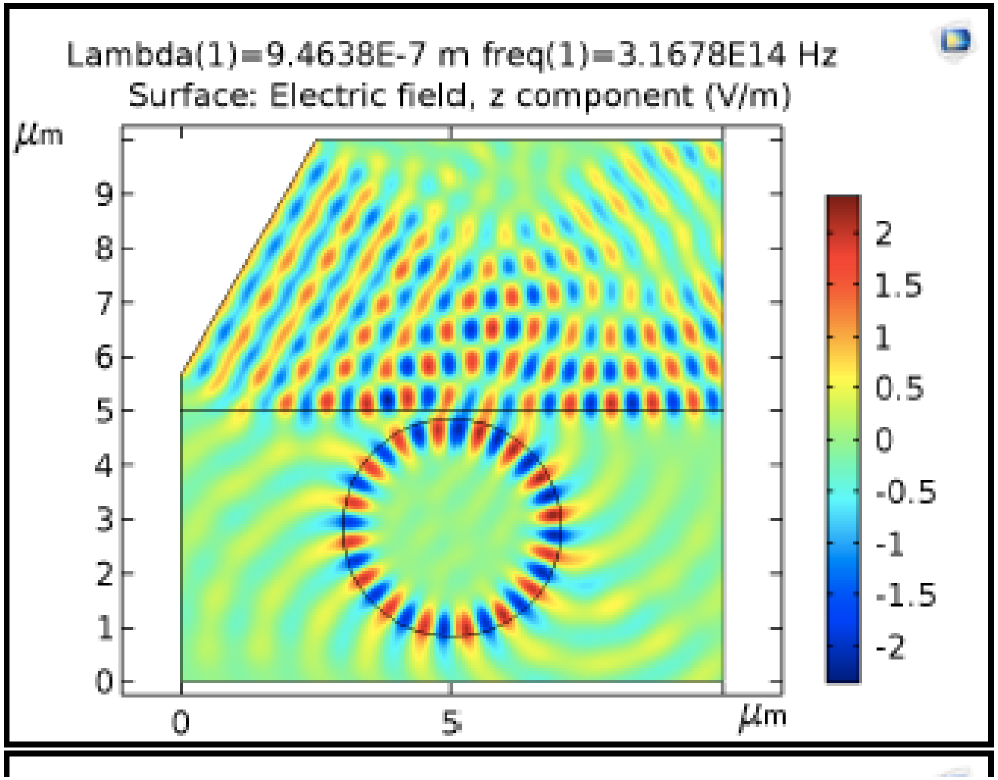


Figure 7.6: Simulation in Mathematica. l=20. The value $j_l(kr) \cdot Re\left(e^{i \cdot m \cdot atan[\frac{y}{x}]}\right)$

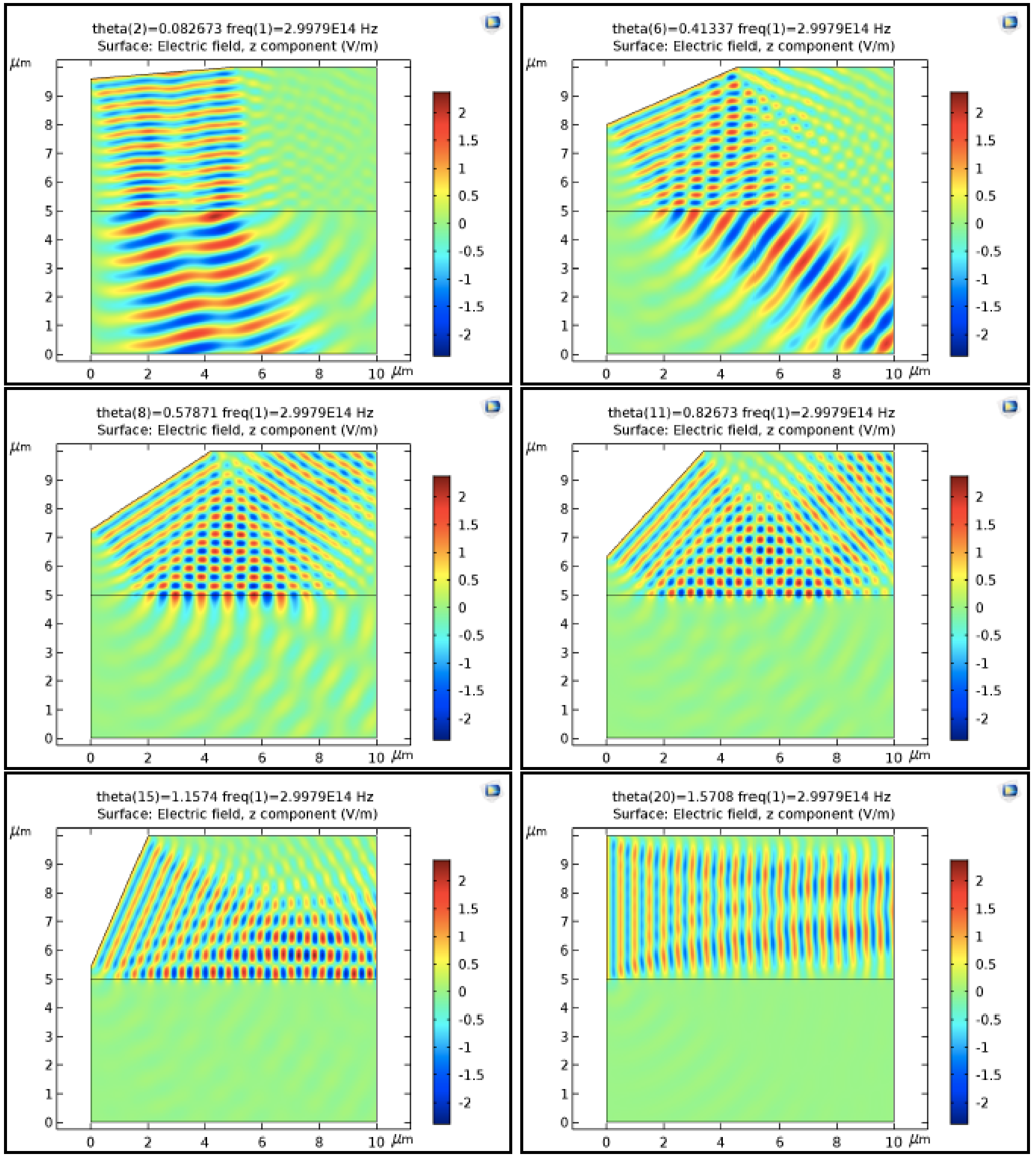


Mathematica simulācija

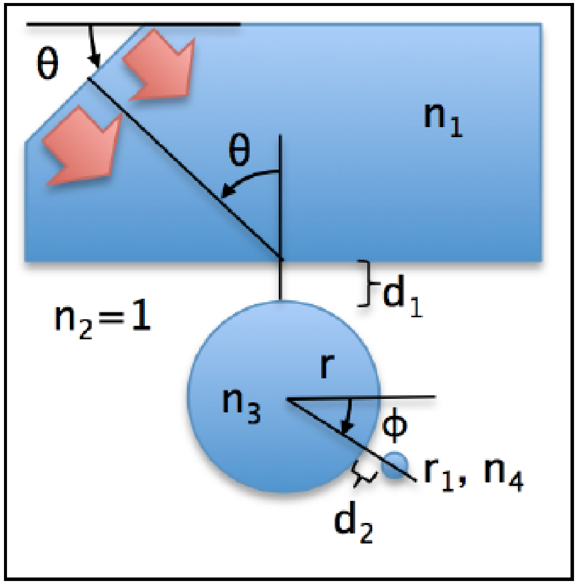
Comsol simulācija

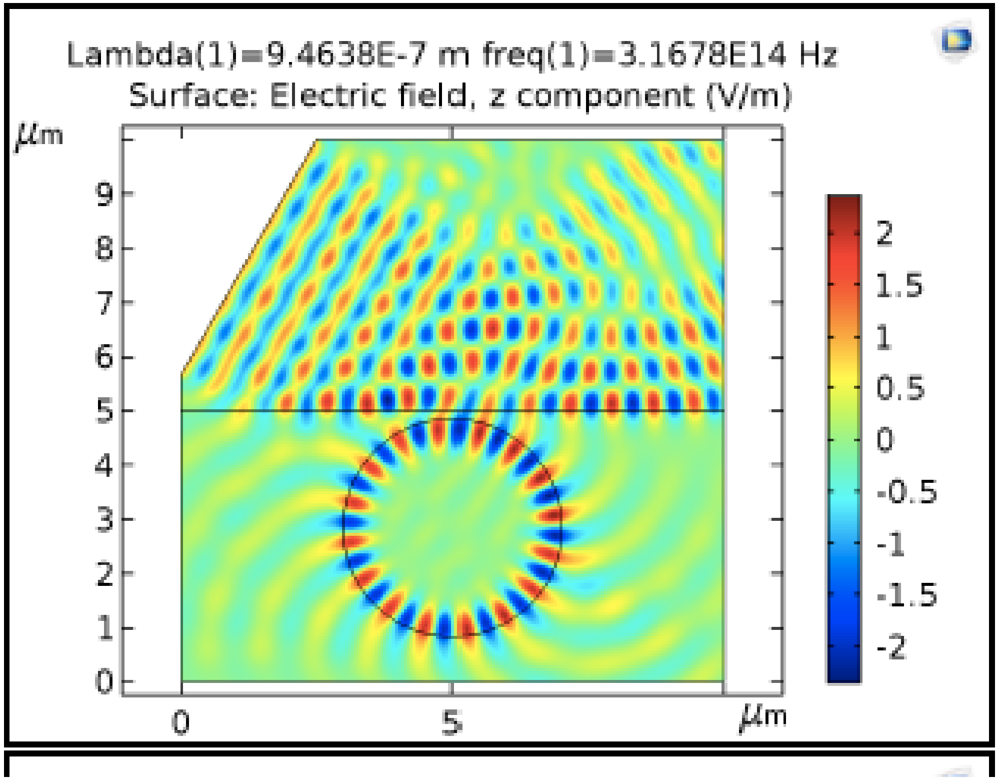
Modelēšana COMSOL Programmā

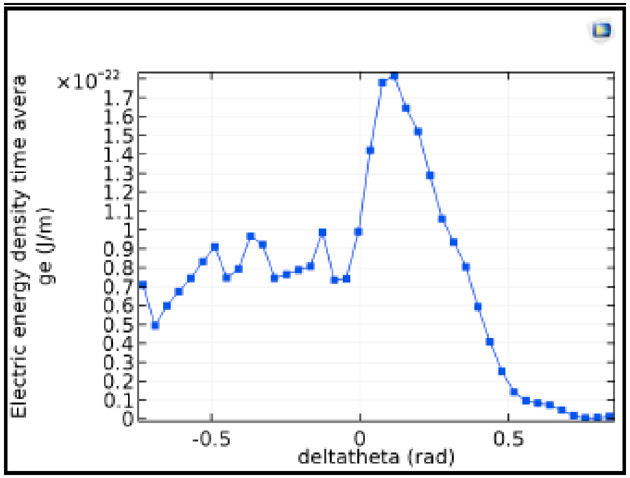
Atkarība no krišanas leņķa (bez rezonatora)



Atkarība no leņķa (ar rezonatoru)

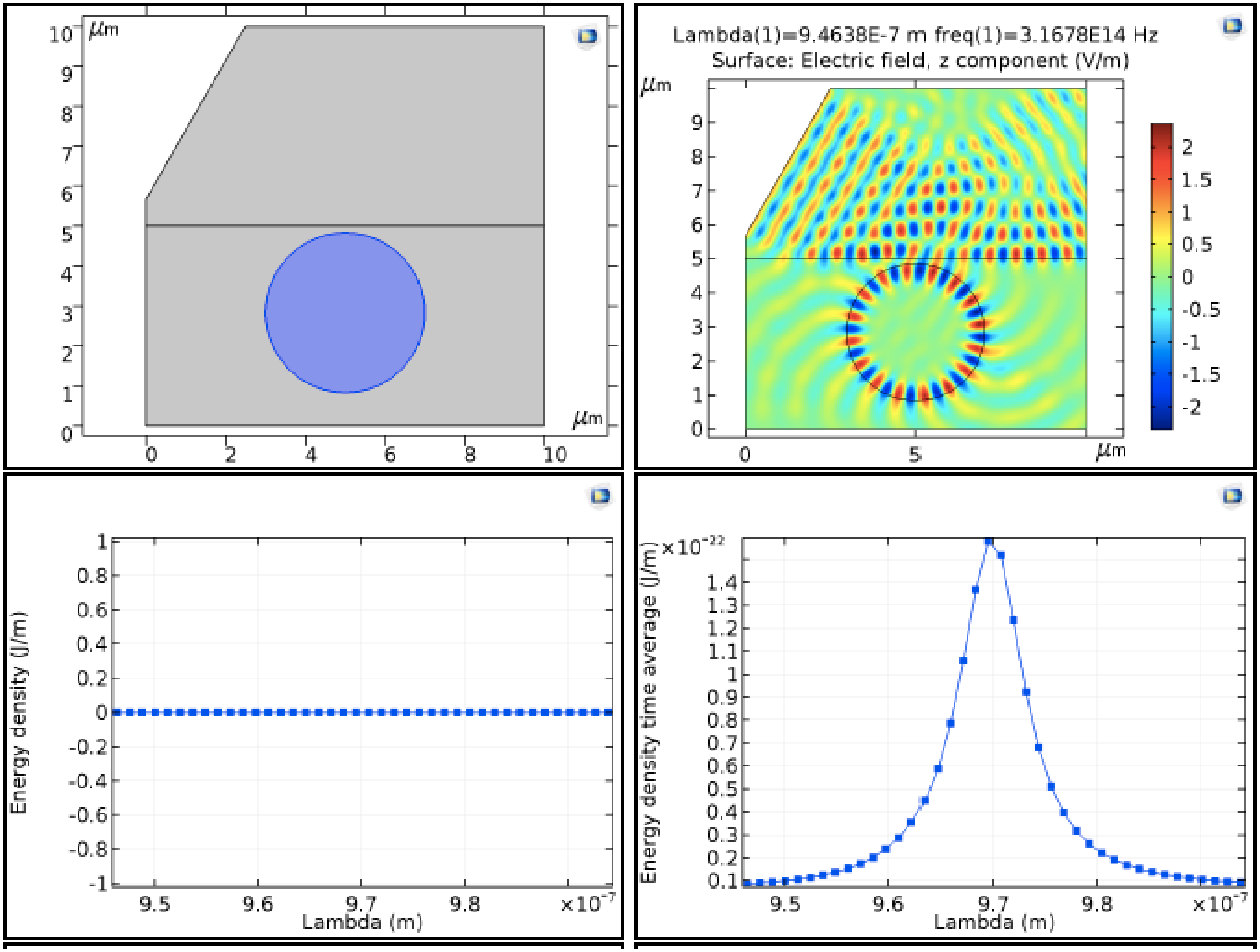


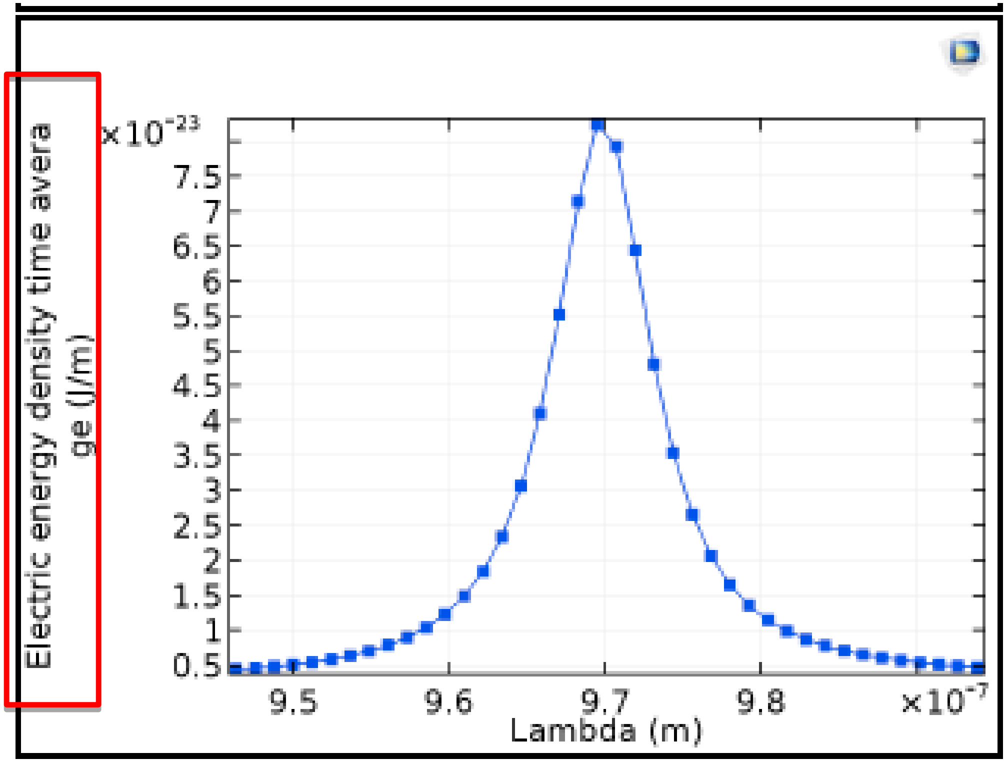


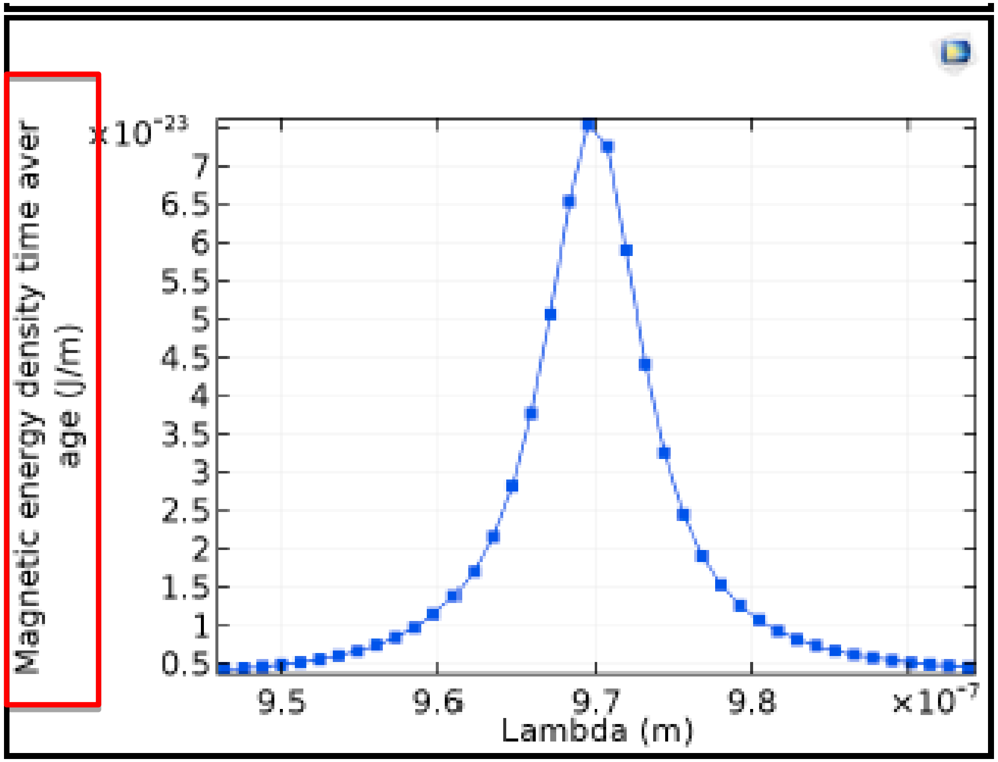


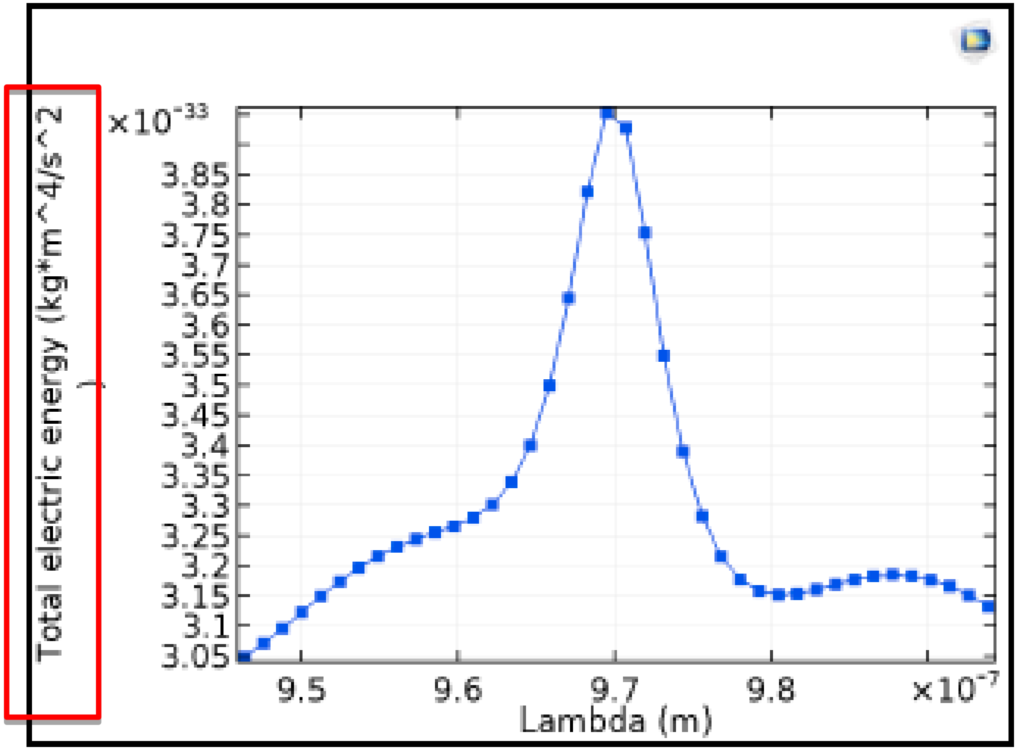
Δθ=0 – pilnīgās iekšējās atstarošanās leņķis

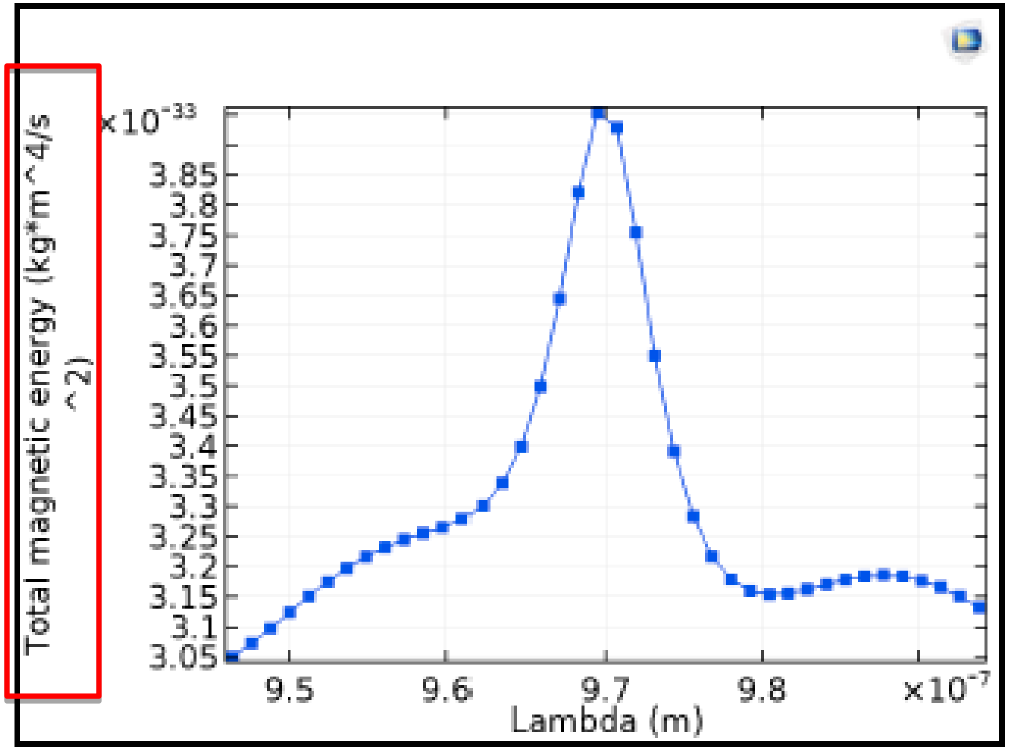
Rezonances atrašana, mainot viļņa garumu



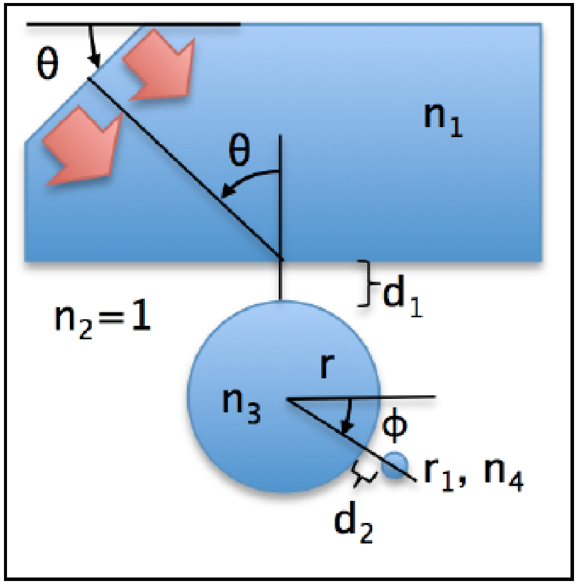

Electric energy density time average (J/m)
×10^-23
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
9.5
9.6
9.7
9.8
×10^-7
Lambda (m)

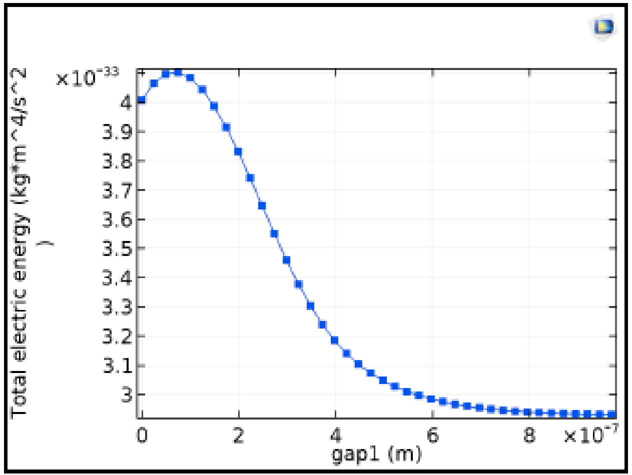

Magnetic energy density time average (J/m)
×10^-23
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
9.5
9.6
9.7
9.8
×10^-7
Lambda (m)

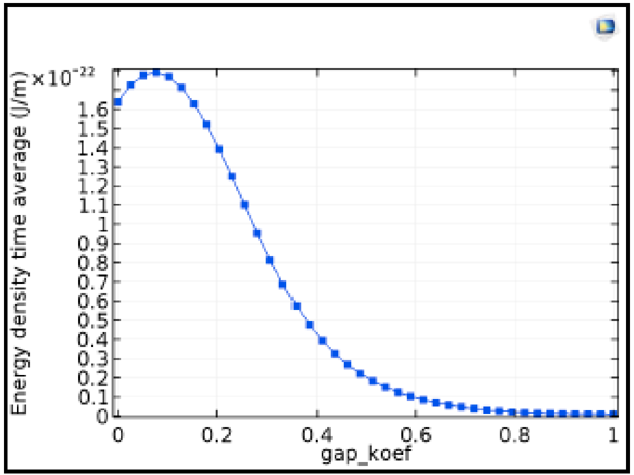

Total electric energy (kg*m^4/s^2)
×10^-33
3.85
3.8
3.75
3.7
3.65
3.6
3.55
3.5
3.45
3.4
3.35
3.3
3.25
3.2
3.15
3.1
3.05
9.5
9.6
9.7
9.8
×10^-7
Lambda (m)

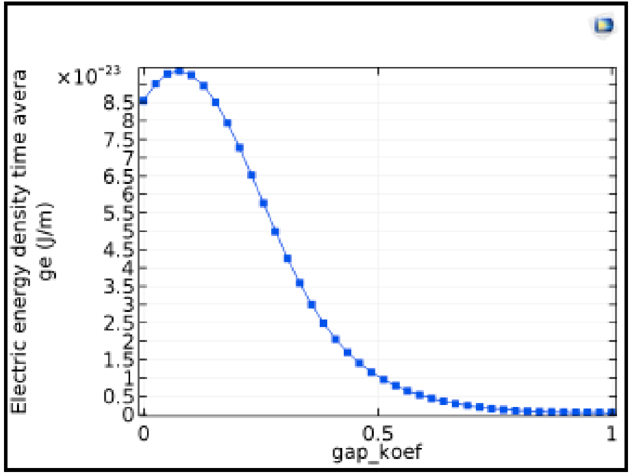

Total magnetic energy (kg*m^4/s^2)
×10^-33
3.85
3.8
3.75
3.7
3.65
3.6
3.55
3.5
3.45
3.4
3.35
3.3
3.25
3.2
3.15
3.1
3.05
9.5
9.6
9.7
9.8
×10^-7
Lambda (m)

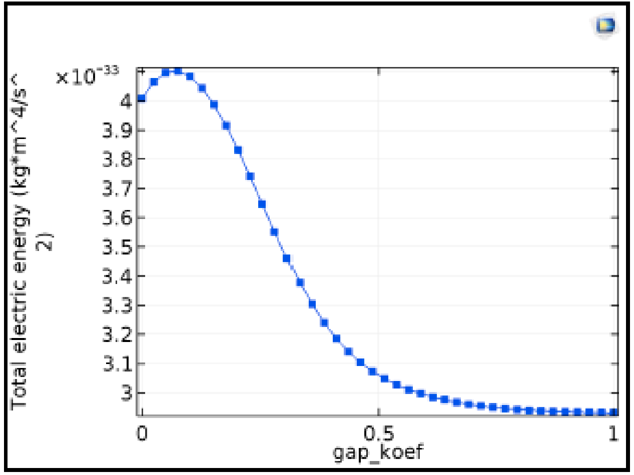
Maina attālumu d_1 no rezonatora līdz prizmai











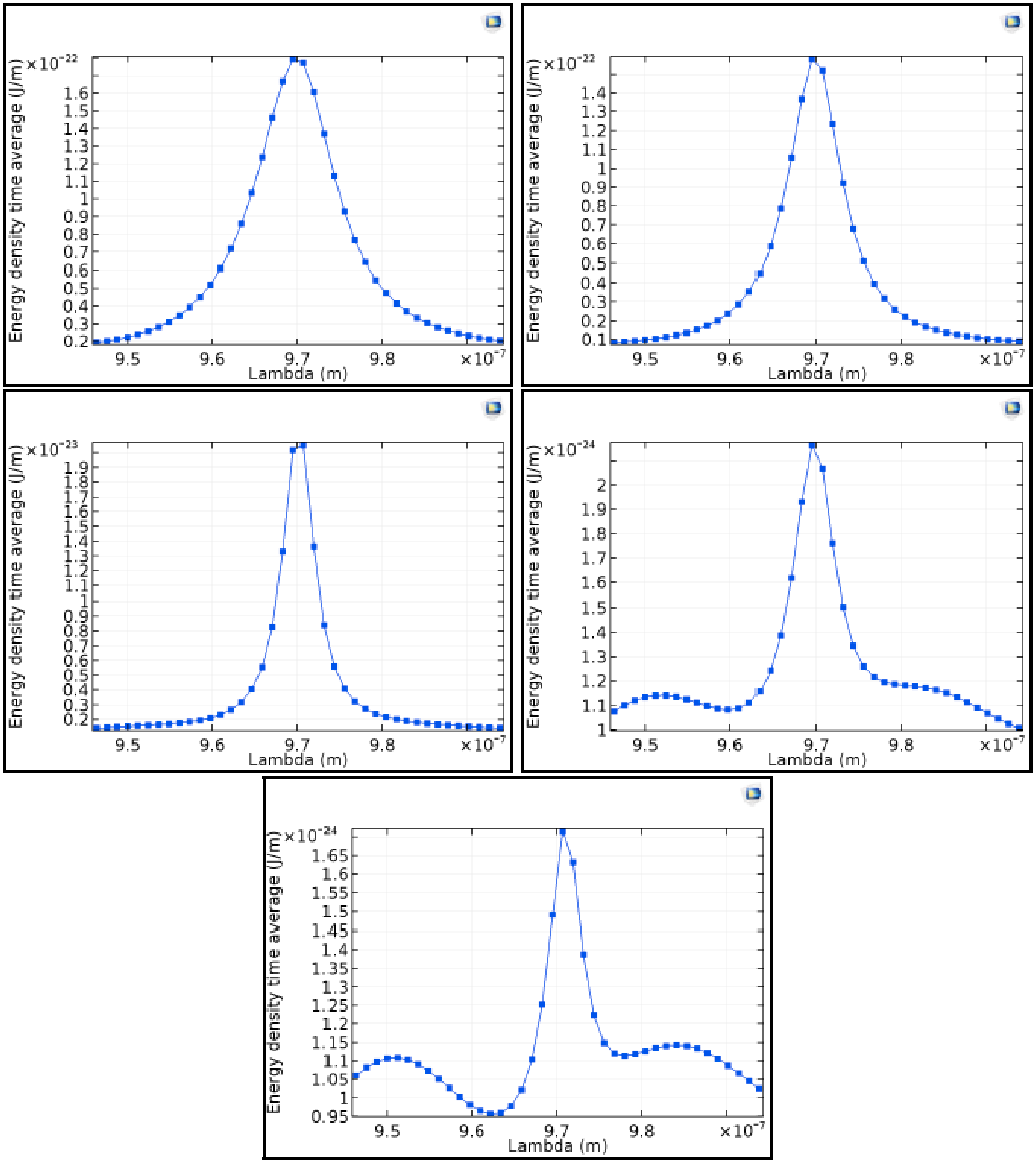


Figure 8.7: Simulation in Comsol Multiphysics. Different wavelength to find the resonance of $N = 16$. Gap between prism and resonator is (a) $\lambda_0/13$; (b) $\lambda_0/6$; (c) $\lambda_0/2$; (d) $0.8\lambda_0$; (e) λ_0.

Rezonators ar “bio-molekulu”

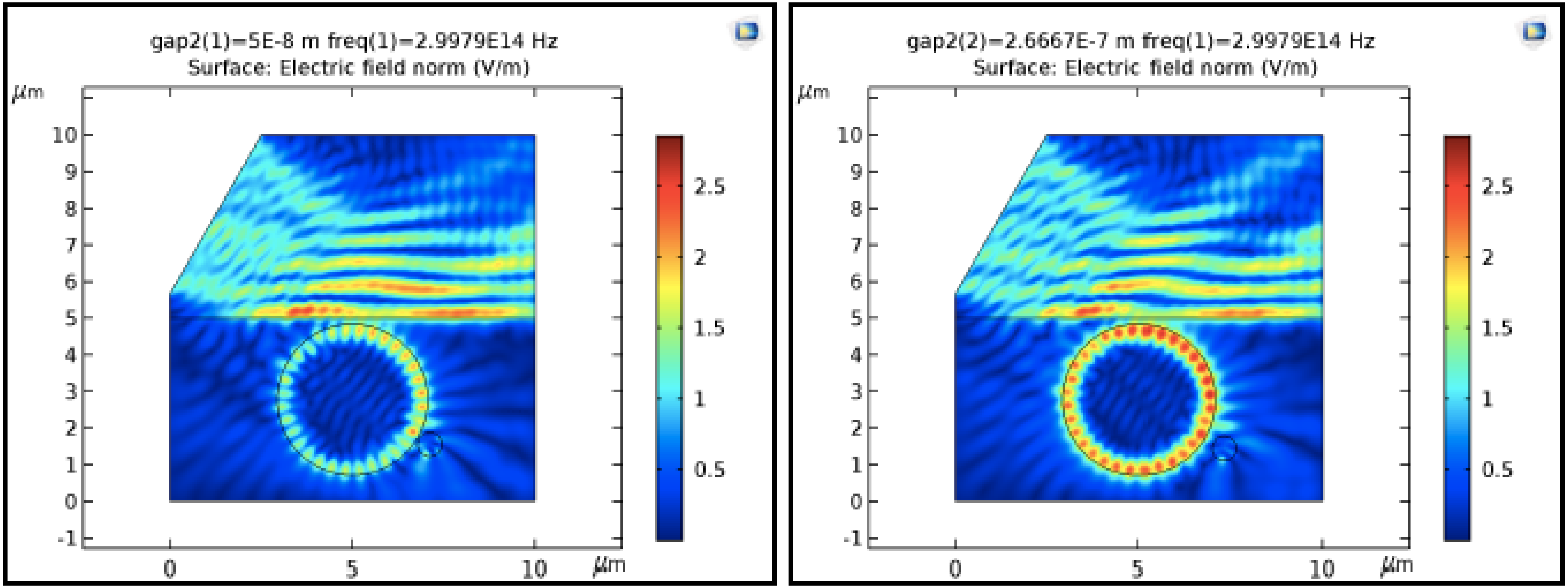
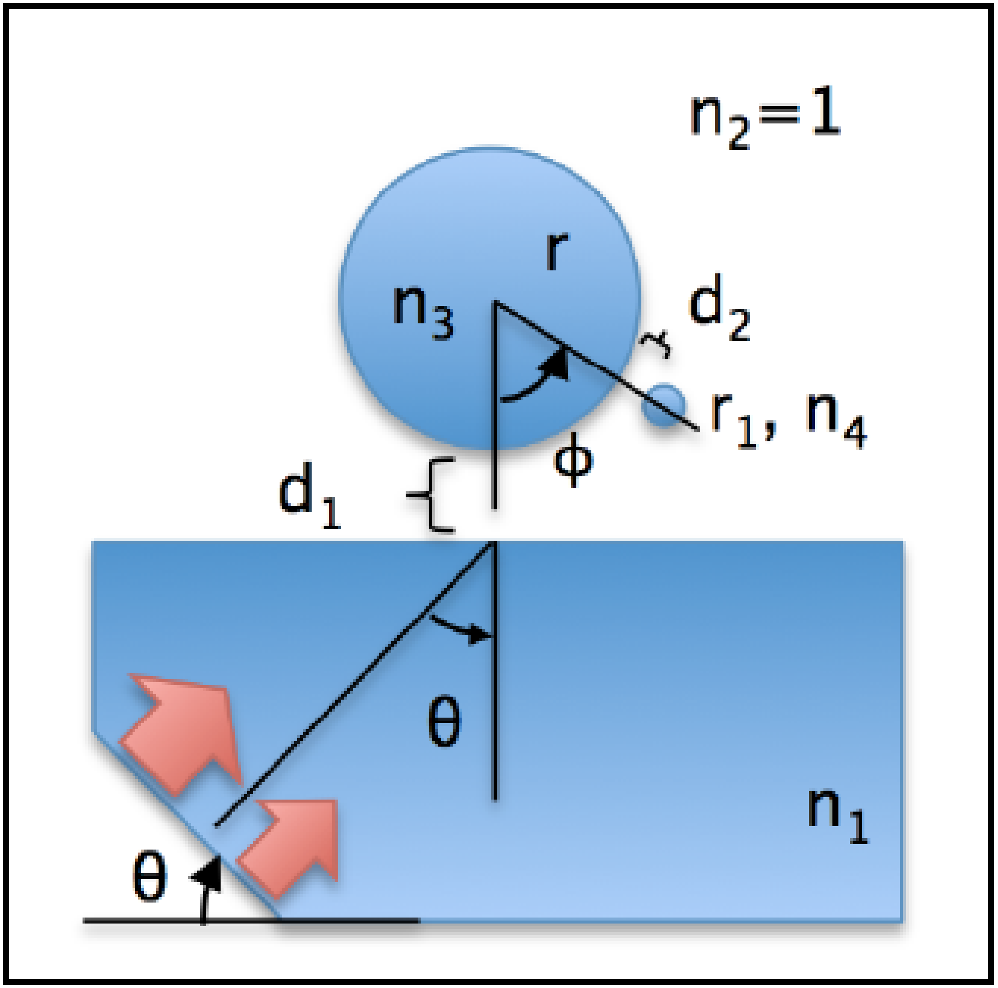
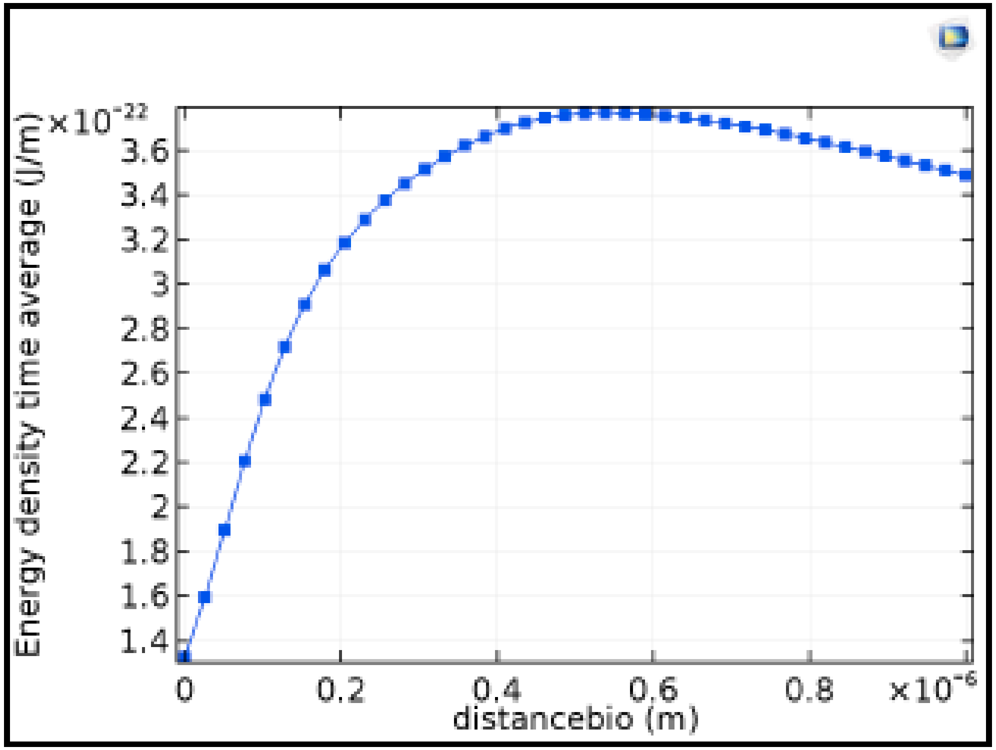


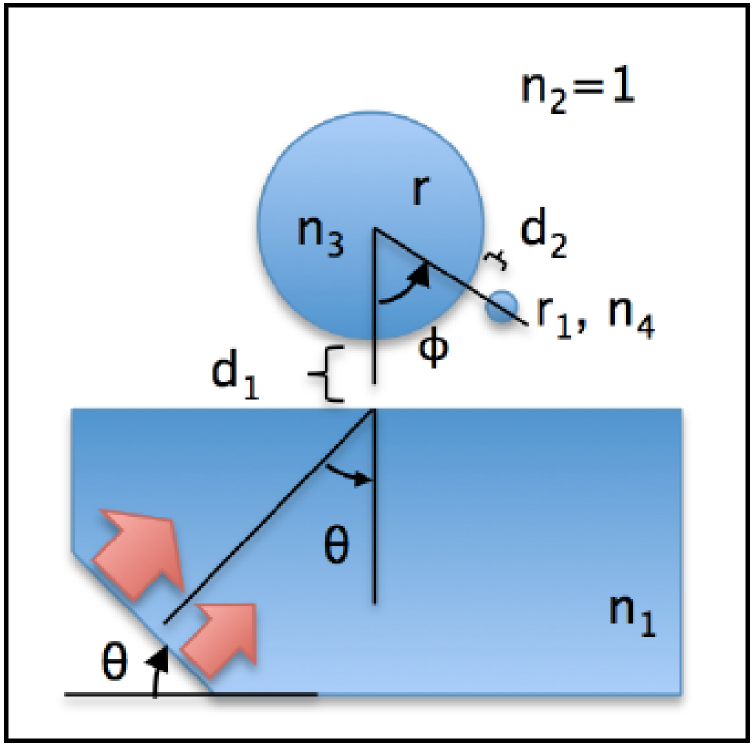
Figure 8.12: Simulation in Comsol Multiphysics. Small angle (left) and appropriate angle (right)

Maina attālumu līdz “bio-molekulai”





Maina biomolekulas novietojumu (fi)



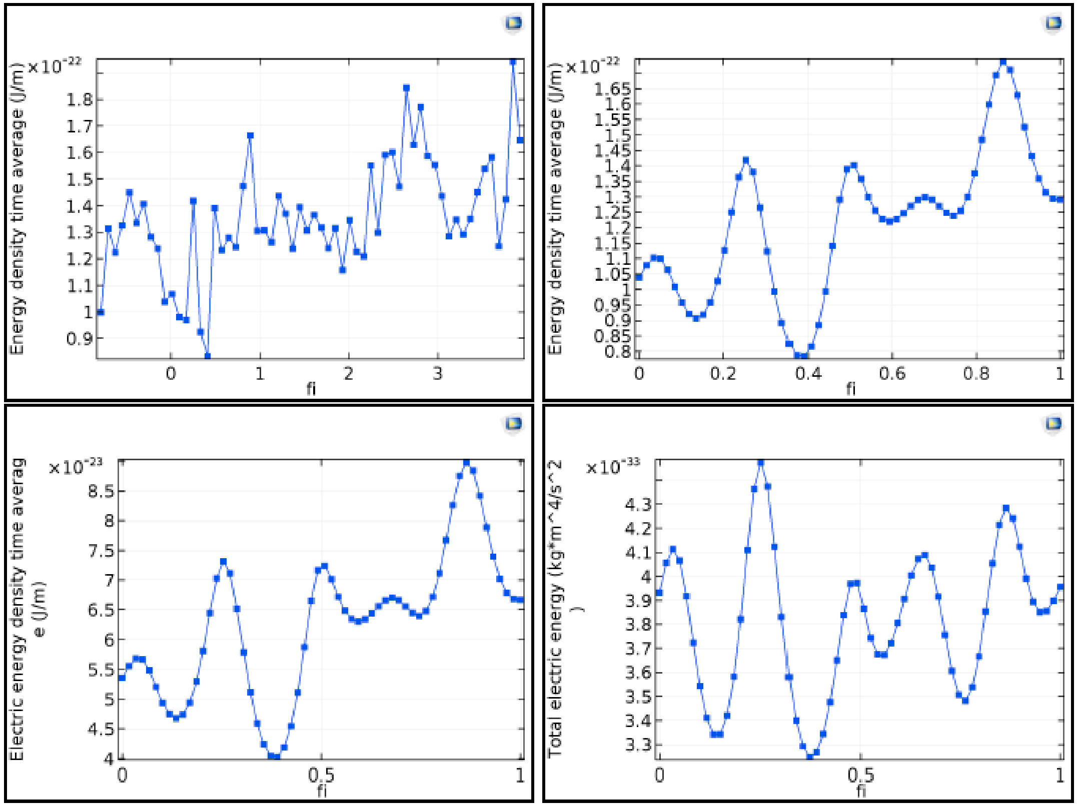


Figure 8.15: Simulation in Comsol Multiphysics. $\lambda_{vacuum} = \lambda_0 = 969.52$ nm, $N = 16$. gap $= \lambda_0/13$, $n = 1.5$, $n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, distancebio $= 0 * \lambda_0$

Bez “bio-molekulas”

Maina biomolekulas izmērus (r)

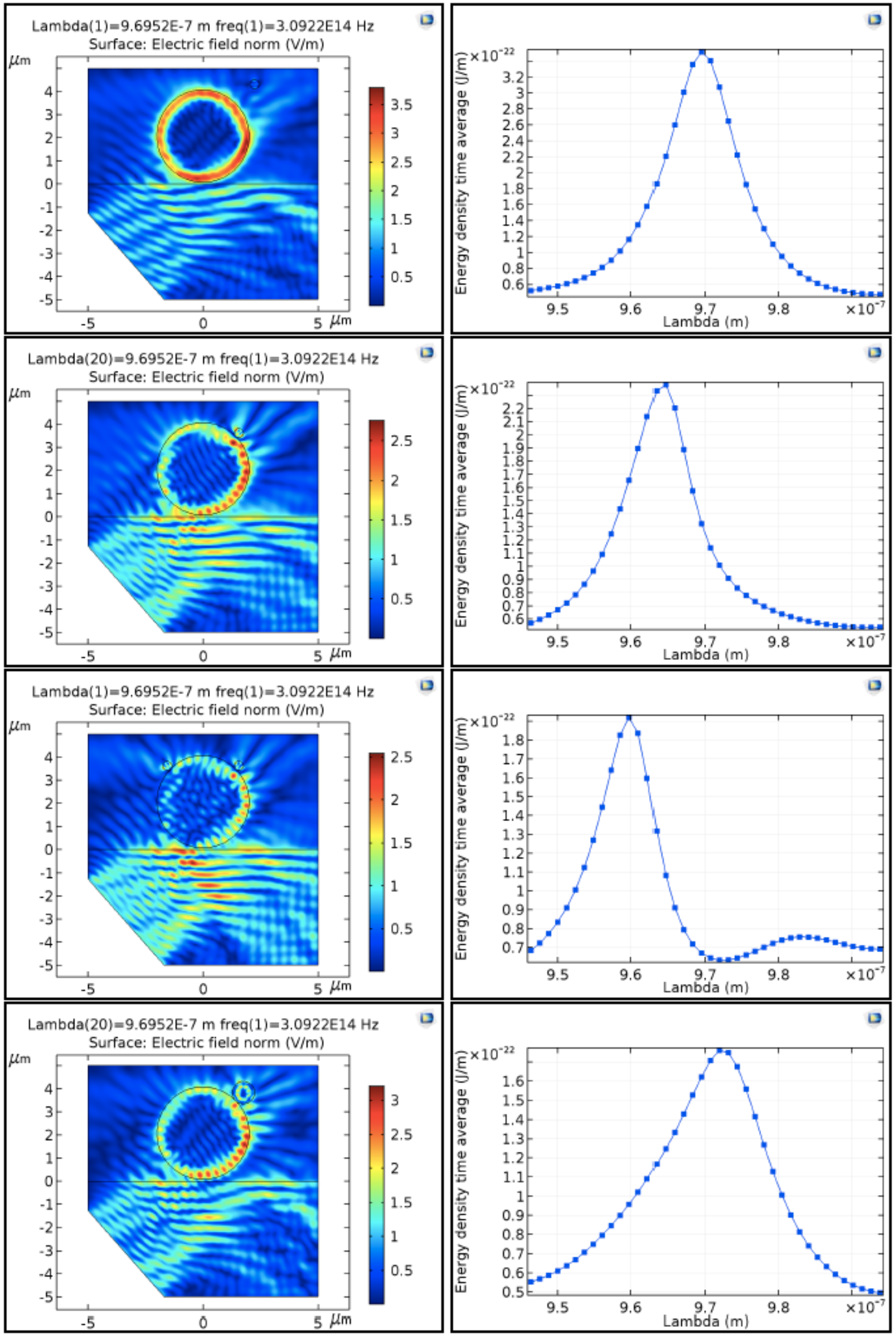


Figure 8.16: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, (a) d_2=λ_0, $r_2 = 0.2\ \mu m$ (b) d_2=0, $r_2 = 0.2\ \mu m$ (c) d_{21}=0, $\phi_1 = 3\pi/4$, d_{22}=0, $\phi_2 = 5\pi/4$, (d) d_2=0, $r_2 \approx \lambda_0$ =969.52 nm

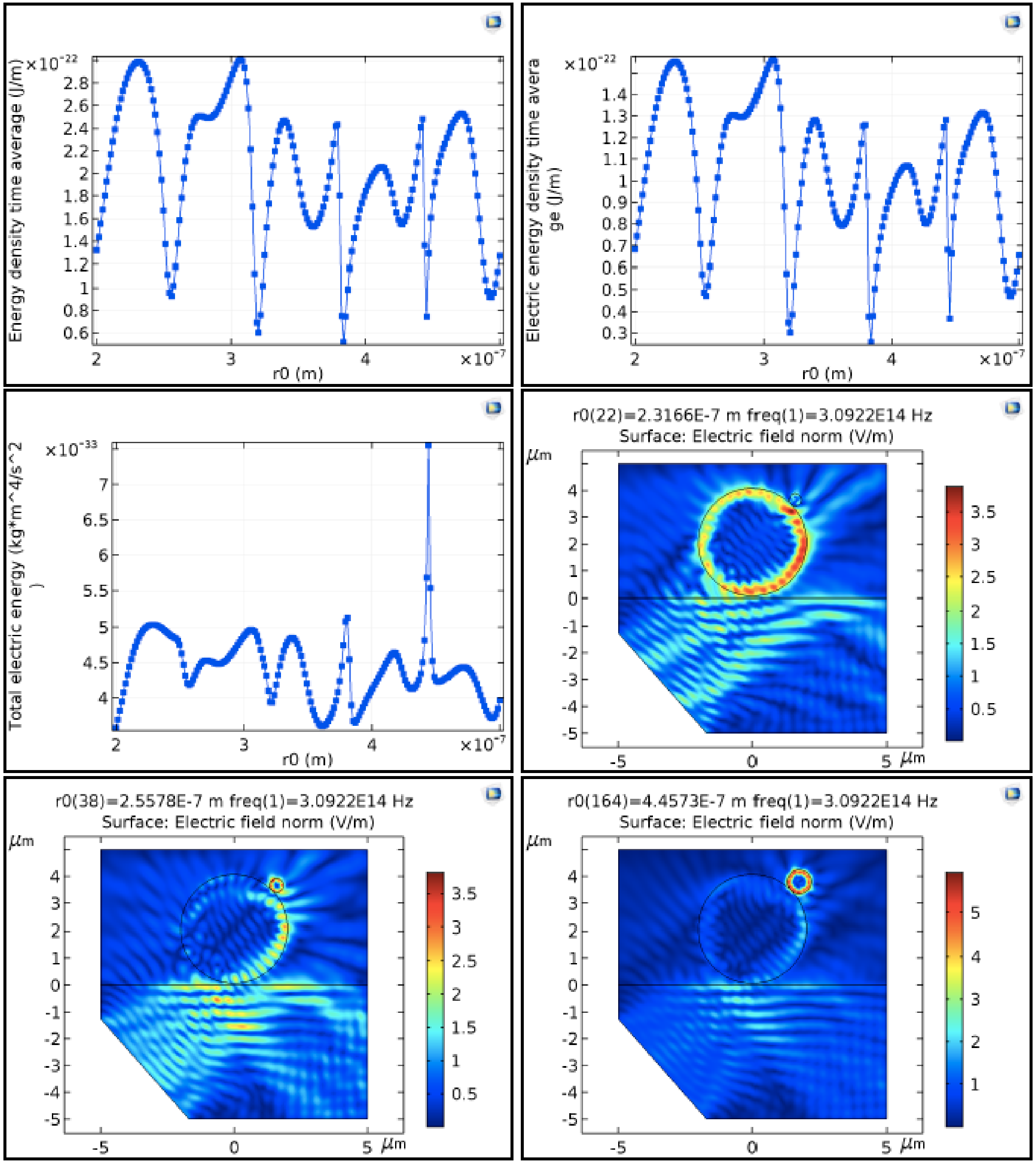


Figure 8.17: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, d_2=0, r_1 is changed (in horizontal axes as r_0). Plots for some postions of $r_1 = r_0$ of resonances

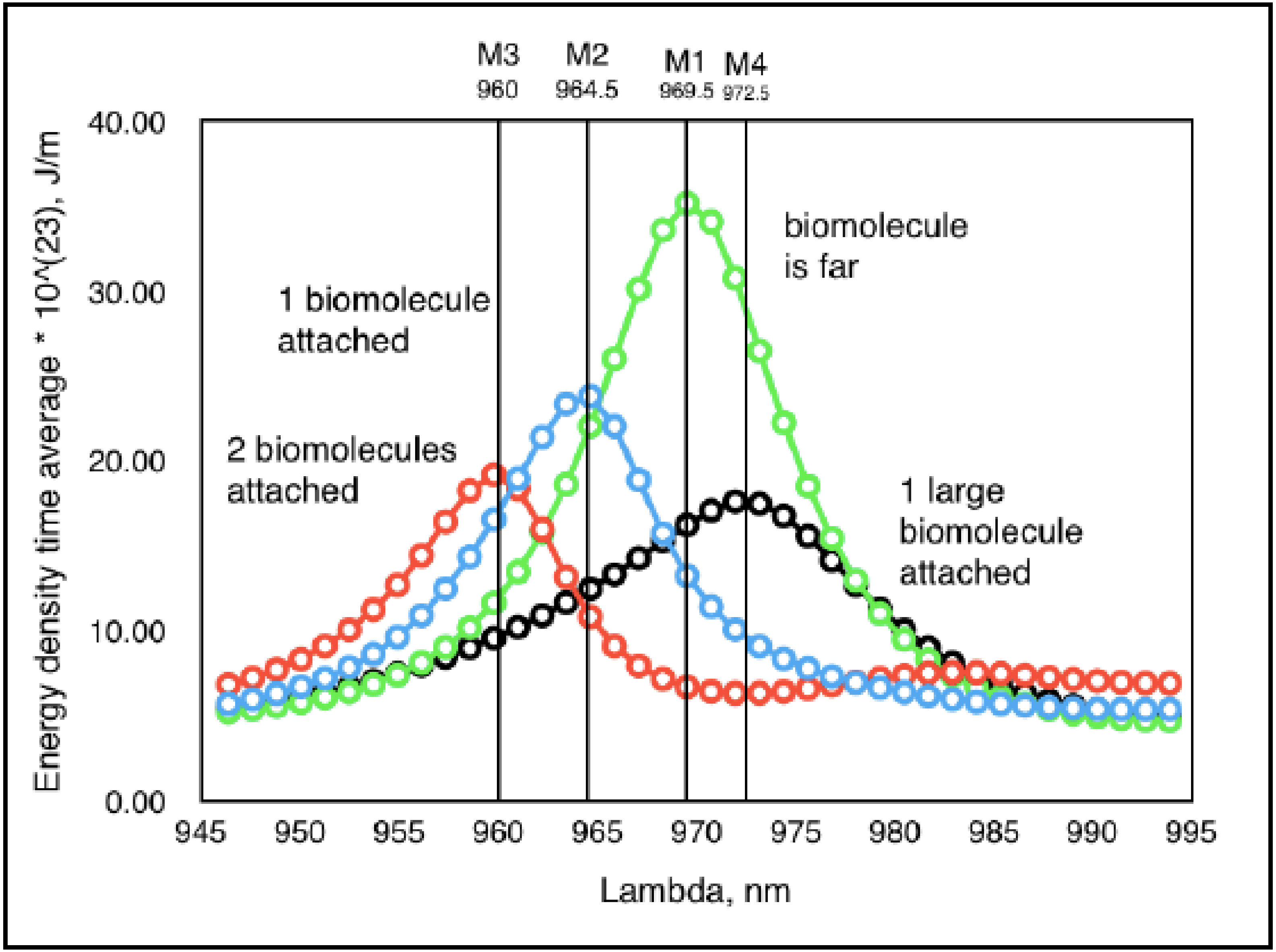


Figure 8.18: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, (M1) $d_2{=}\lambda_0$, $r_2 = 0.2\ \mu m$ (M2) $d_2{=}0$, $r_2 = 0.2\ \mu m$ (M3) $d_{21}{=}0$, $\phi_1 = 3\pi/4$, $d_{22}{=}0$, $\phi_2 = 5\pi/4$, (M4) $d_2{=}0$, $r_2 = 969.52$ nm $\approx \lambda_0$

Nesfēriska biomolekula

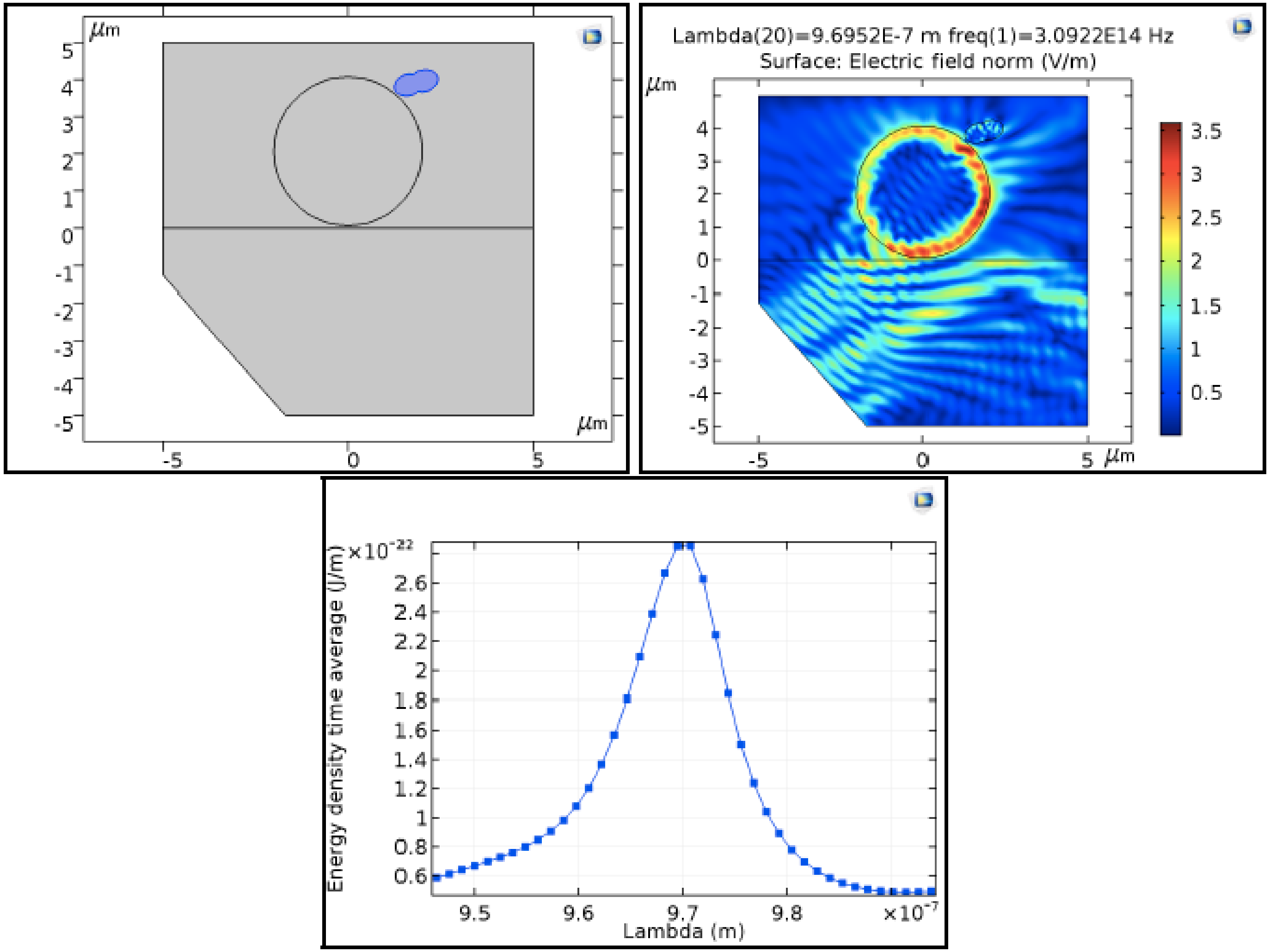


Figure 8.19: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, d_2=0, r_1 is changed (in horizontal axes as r_0). Lambda is a bit shifter to larger wavelengths compared to non-bio environment

Vairākas biomolekulas

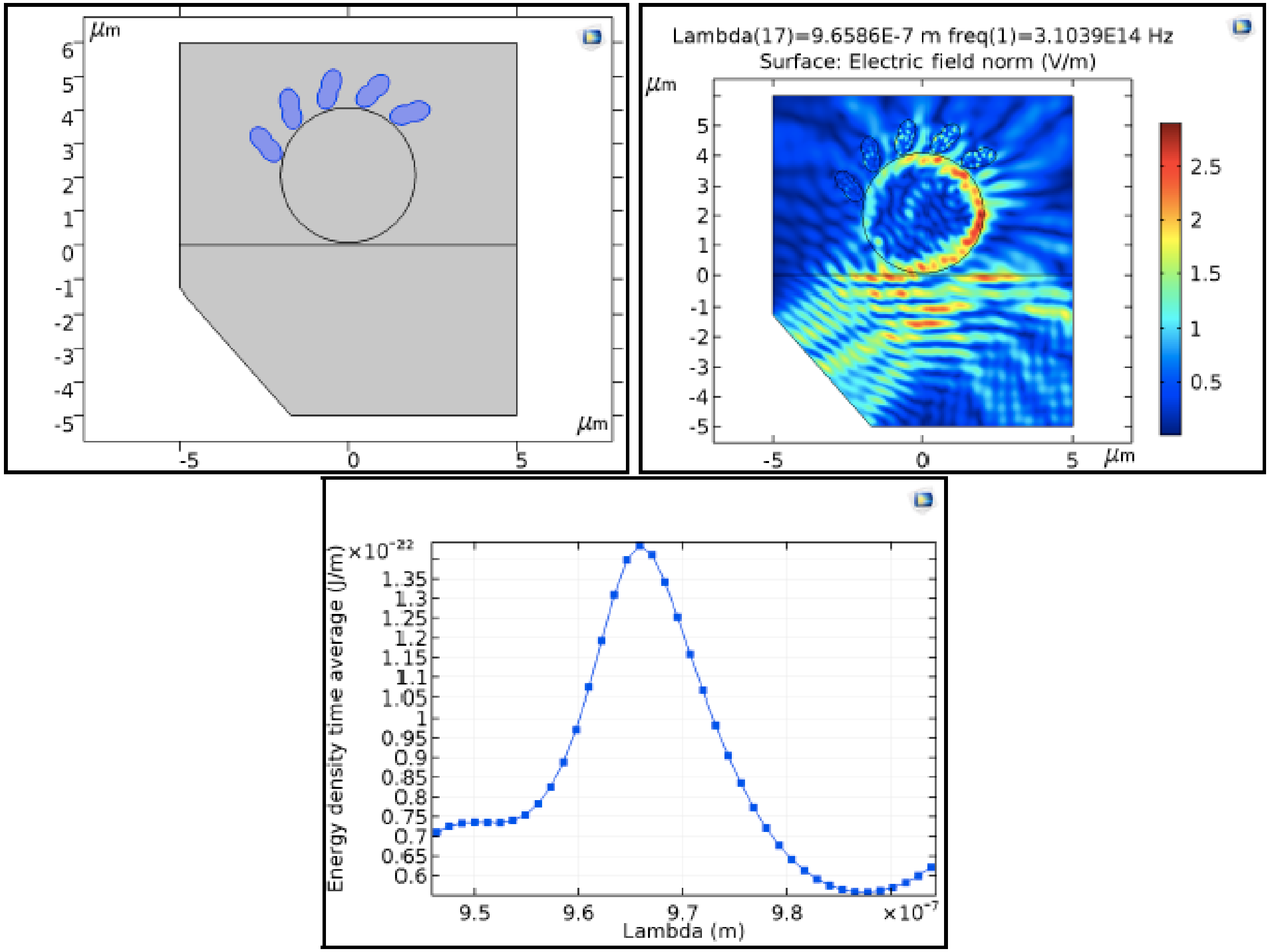


Figure 8.20: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, d_2=0, r_1 is changed (in horizontal axes as r_0).

Signāla atkarība no rezonatorā ietilpstošo viļņu garumu skaita N

N=16

N=32

N=64

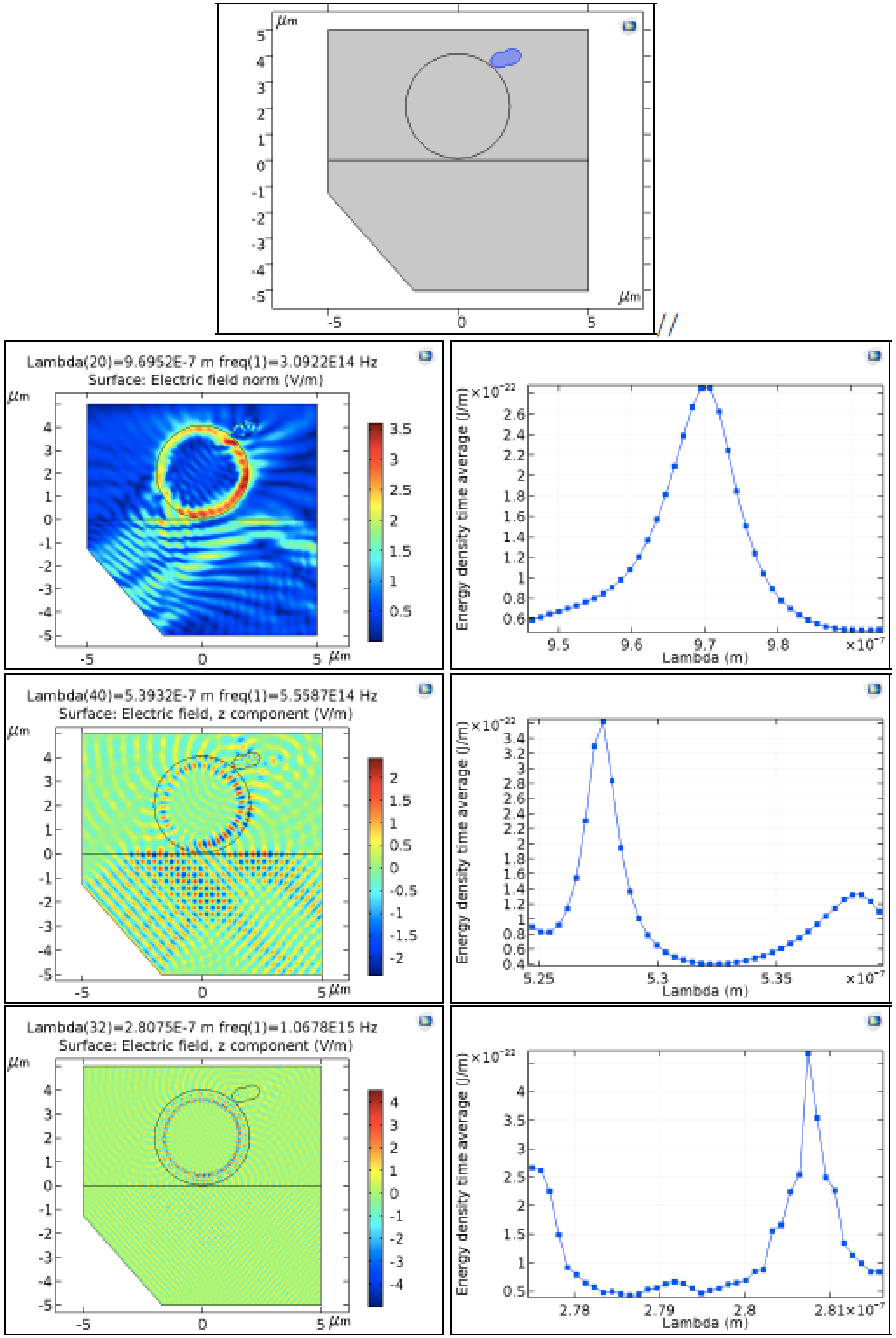


Figure 8.21: Simulation in Comsol Multiphysics. $n_1 = n_3 = 1.5$, $n_2 = 1$, $n_4 = n_{bio} = 3$, $r = 2\ \mu m$, $\theta_{light} = 0.846$ rad, $d_1 = \lambda_0/13$, $\phi = 3pi/4$, d_2=0, r_1 is changed (in horizontal axes as r_0). Lambda is a bit shifter to larger wavelengths compared to non-bio environment (A) N=16, (b) N=32 ; (c) N=64

Secinājums:
Noteiktās robežās pieaugot N, rezonanšu platums samazinās

Jaunu čukstošās galerijas modu mikrorezonatoru izstrāde optisko frekvenču standartu un biosensoru pielietojumiem, un to raksturošana ar femtosekunžu optisko frekvenču ķemmi

Otrā ceturkšņa atskaite

4. Biosensori

NACIONĀLAIS ATTĪSTĪBAS PLĀNS 2020

EIROPAS SAVIENĪBA
Eiropas Reģionālās attīstības fonds

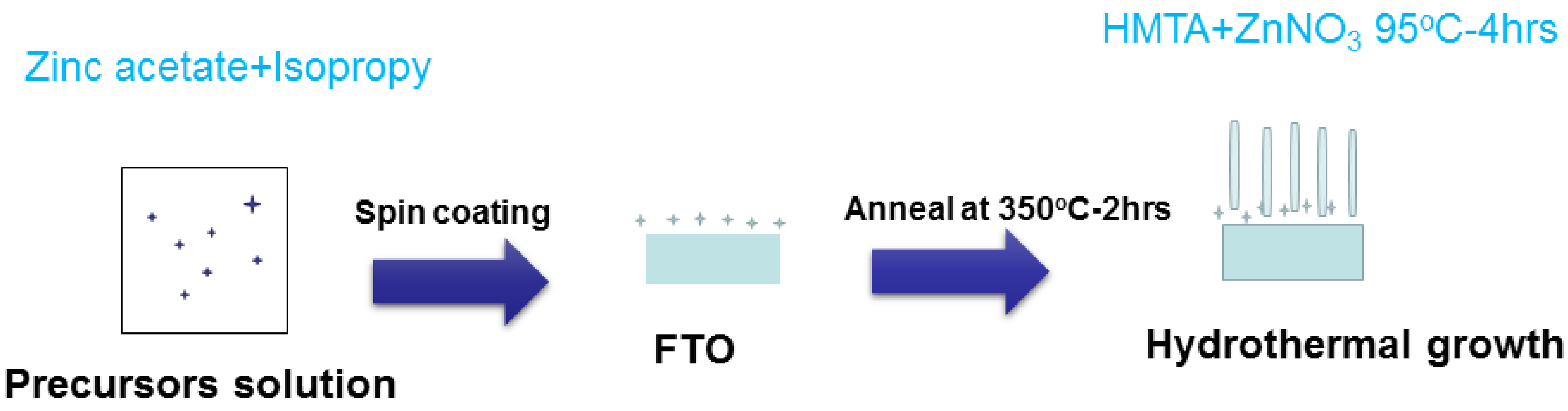
IEGULDĪJUMS TAVĀ NĀKOTNĒ

ERAF projekts Nr. 1.1.1.1/16/A/259

Hydrothermal Synthesis of ZnO

Step 1: **Plasma cleaning of FTO glass**

- Deionised water-isopropanol



Zinc acetate+Isopropy

HMTA+$ZnNO_3$ 95°C-4hrs

Spin coating

Anneal at 350°C-2hrs

Precursors solution

FTO

Hydrothermal growth

The schematic illustration of fabrication process of the ZnO nanowire arrays on the FTO glass substrate

Structure Characterisation

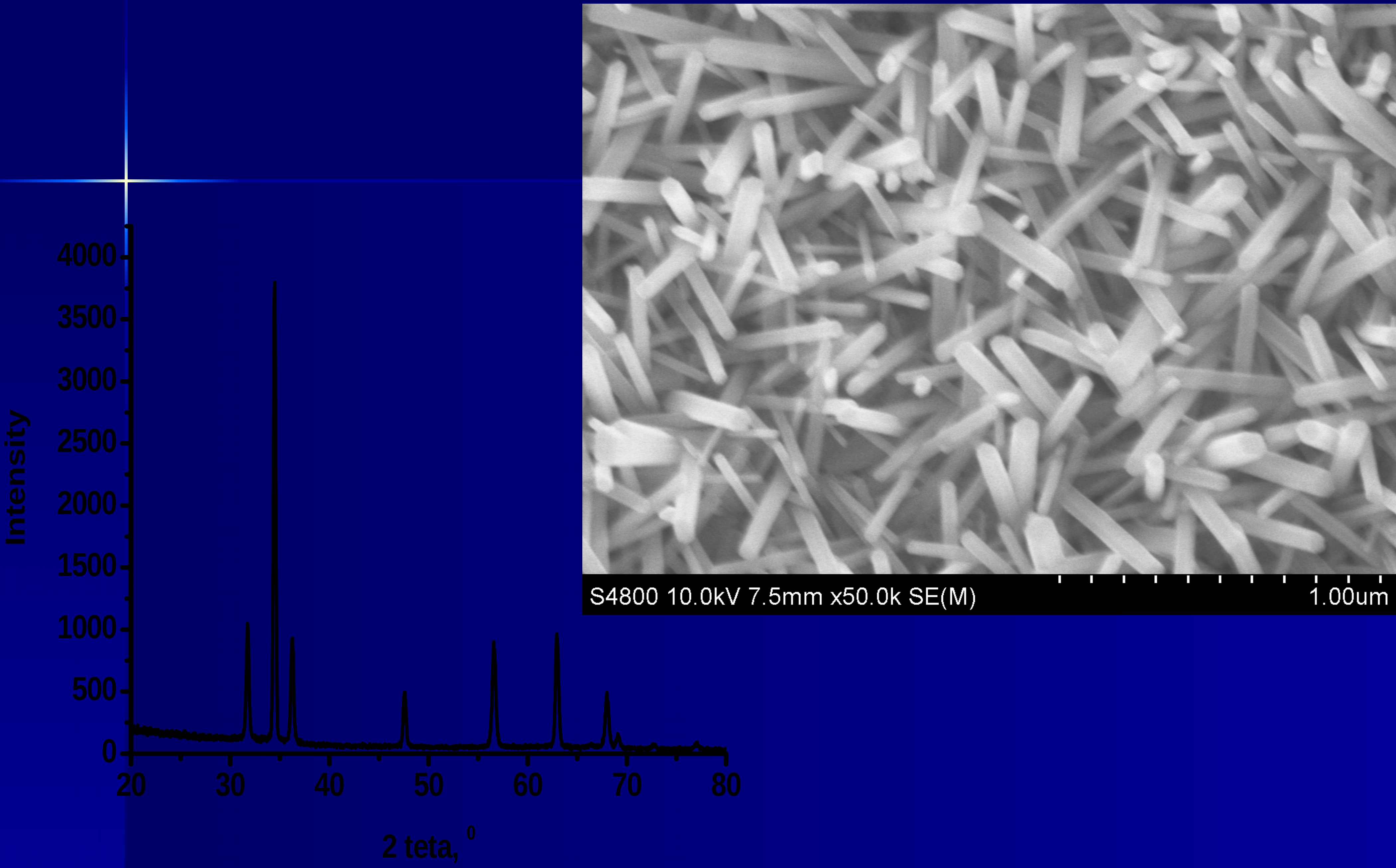


Figure 1: XRD (on the left), SEM (on the right) of ZnONRs

TEM

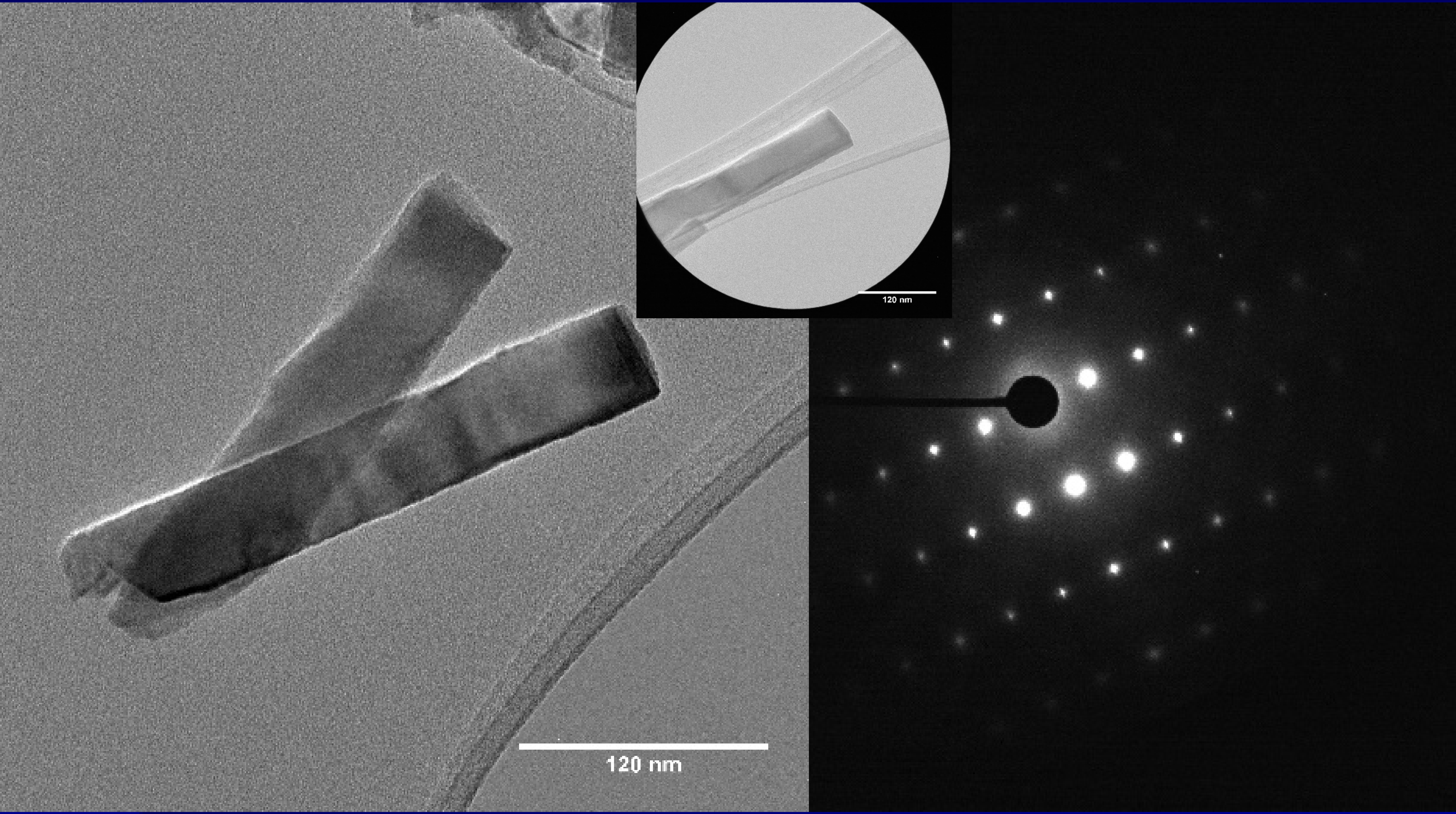


Figure 2: TEM image and electron diffration

Next steps

- Deposition of thin metal oxide films over PS microspheres
- Deposition of metal oxide nanolaminates over WGM resonators
- Coating of thin layers on internal surfaces of quartz capillaries
- Fabrication of microresonators using lithography
- Structure and optical characterization
- Preparation of paper draft on new coatings for WGM biosensors