We consider an elliptic equation with nonlinear and nonlocal boundary conditions, which arises in conductive-radiative heat transfer problems, see, for instance, [1; 2; 3]. The corresponding to our problem variational equality reads as

\[
\int_{\Omega} \left[ k_1 \langle \nabla (u + u_*), \nabla \eta \rangle + k_2 (u + u_*) \eta \right] \, dx + \int_{\Gamma} \sigma [ (I - H) (| u + u_* |^3 (u + u_*))] \eta \, dS = \int_{\Omega} \langle f, \eta \rangle \, dx + \int_{\Gamma} g \eta \, dS \quad \forall \eta \in V,
\]

(1)

where \( \Omega = \Sigma \times [0, L] \subset \mathbb{R}^3 \) is a bounded cylinder, \( V \) is a subspace of \( W_2^1 (\Omega) \) of functions that are zero on the intersection of \( \Omega \) with the plane \( \{ x_3 = 0 \} \), \( \Gamma \) is the lateral surface of \( \Omega \), \( k_1, k_2, \sigma \) are positive constants, but \( H \) is a nonlocal bounded linear operator from \( L_p (\Gamma) \) to \( L_p (\Gamma) \) such that for \( p = 1 \) its norm is less than 1.

We show that there exists a two level iterative process that converges to the solution of (1). The first level consists of the Newton-type process

\[
\int_{\Omega} \left[ k_1 \langle \nabla v_{k+1} + u_*, \nabla \eta \rangle + k_2 (v_{k+1} + u_*) \eta \right] \, dx + \int_{\Gamma} \sigma \psi (v_k) v_{k+1} \, dS = \langle \langle F(v_k), \eta \rangle \rangle \quad \forall \eta \in V, \quad k = 1, 2, \ldots,
\]

with appropriate nonnegative function \( \psi \) and \( F(v_k) \in (V)^* \). In its turn, the second level consists on iterations of the type

\[
\int_{\Omega} \left[ k_1 \langle \nabla u_{k+1} + u_*, \nabla \eta \rangle + k_2 (u_{k+1} + u_*) \eta \right] \, dx + \int_{\Gamma} \sigma \left[ | u_{k+1} + u_* |^3 (u_{k+1} + u_*) \right] \eta \, dS = \int_{\Gamma} \sigma H \left[ | u_k + u_* |^3 (u_k + u_*) \right] \eta \, dS + \langle \langle F_0, \eta \rangle \rangle \quad \forall \eta \in V, \quad k = 1, 2, \ldots,
\]

with an appropriate \( F_0 \in (V)^* \).

REFERENCES


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