ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
Abstracts of the 8th Latvian Mathematical Conference, April 9–10, 2010, Valmiera, Latvia
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TOEPLITZ WORDS[1]

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Let Σ be a finite non-empty set of symbols (*letters*), which is called an *alphabet*. Let Σ^* (respectively, Σ^ω) denote the set of all *finite* (respectively, *infinite*) words over alphabet Σ. For a deeper coverage of the topic see, e.g., [1; 2]. Let *?* be a special letter or hole not in Σ. For a word $x \in (\Sigma \cup \{?\})^\omega$ let $T^0(x) = x$, and $\forall i \geq 0$ $T^{i+1}(x)$ is obtained from $T^i(x)$ by replacing sequentially all occurrences of the letter *?* by letters of a word $x \in (\Sigma \cup \{?\})^\omega$. The limit of the sequence

$$T^\omega(x) = lim_{i\to\infty} T^i(x) \tag{1}$$

is called *Toeplitz word*, cf. [3]. We consider cases when Toeplitz words are generated from non-periodic infinite words. Special significance is paid to Toeplitz transformation on Thue-Morse word: The *Thue-Morse* word over alphabet $\Sigma = \{0, 1\}$ is defined as the limit

$$\tau^\omega = lim_{n\to\infty} \tau_0^n, \tag{2}$$

where the sequences of words $(\tau_0^n)_{n\geq 0}$, $(\tau_1^n)_{n\geq 0}$ are defined as follows, cf. [2]:

$$\begin{gathered} \tau_0^0 = 0, \tau_1^0 = 1, \\ \forall n \geq 0 (\tau_0^{n+1} = \tau_0^n \tau_1^n \wedge \tau_1^{n+1} = \tau_1^n \tau_0^n). \end{gathered} \tag{3}$$

We prove that after finite number of iterations Thue-Morse word over alphabet $\{0, ?\}$ is transferred into Thue-Morse word over another alphabet. After infinitely many iterations it is transferred into periodic word 0^ω. Further we consider Toeplitz transformation on bounded and finitely generated bi-ideals, cf. [4].

REFERENCES

[1] J-P. Allouche and J. Shallit. *Automatic Sequences.* Cambridge University Press, 2003.

[2] M. Lothaire. *Algebraic Combinatorics on Words.* Cambridge University Press, 2002.

[3] J. Cassaigne and J. Karhumäki. Toeplitz Words, Generalized Periodicity and Periodically Iterated Morphisms. *European Journal of Combinatorics*, **18** (5):497-510, 1997.

[4] J. Buls and A. Lorencs. From Bi-ideals to Periodicity. In: , Int. Proc. of the XIth Intern. conference Mons days of Theoretical Computer Science, IFSIC/IRISA, Rennes, France, 2006.p.97-110

[1] This work was partially supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008