Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

NUMERICAL SOLUTION OF FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

MIKK VIKERPUUR, ARVET PEDAS, ENN TAMME

Institute of Mathematics, University of Tartu

Liivi 2, Tartu 50409, Estonia

E-mail: azzo@ut.ee, arvet.pedas@ut.ee, enn.tamme@ut.ee

We consider a class of boundary value problems for fractional integro-differential equations

$$(D_*^\alpha y)(t) + h(t)y(t) + \int_0^t (t-s)^{-\kappa} K(t,s)y(s)ds = f(t), \quad 0 \le t \le b, \quad 0 < \alpha < 1, \quad 0 \le \kappa < 1, \tag{1}$$

$$\gamma_0 y(0) + \gamma_1 y(b_1) = \gamma, \ 0 < b_1 \le b, \ \gamma_0, \gamma_1, \gamma \in (-\infty, \infty), \tag{2}$$

where h, f and K are some given continuous functions on $[0,b]$ and $\{(t,s) : 0 \le s \le t \le b\}$, respectively. In (1) $D_*^\alpha y$ is the Caputo fractional derivative of y of order $\alpha \in (0,1)$ defined by

$$(D_*^\alpha y)(t) = \frac{d}{dt}\left(J^{1-\alpha}[y - y(0)]\right)(t), \quad 0 \le t \le b,$$

with

$$(J^\beta y)(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1}\, y(s)\, ds, \quad t > 0, \quad \beta > 0,$$

where Γ is the Euler gamma function.

Using an integral equation reformulation of (1)-(2), first some regularity properties of the exact solution of (1)-(2) are studied. Using the obtained information about the possible singular behavior of the exact solution and spline collocation techniques, the numerical solution of (1)-(2) by suitable non-polynomial approximations is considered. The attainable order of global convergence of the proposed algorithms is discussed and a superconvergence result for a special choice of grid and collocation parameters is given. A numerical illustration will also be presented. Our approach is based on some ideas and results of [1].

REFERENCES

[1] A. Pedas, E. Tamme. Piecewise polynomial collocation for linear boundary value problems of fractional differential equations. *J. Comput. Appl. Math.*, **236** (13):3349–3359, 2012.