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REGULARIZING A VOLTERRA INTEGRAL EQUATION OF THE FIRST KIND

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We consider the inverse problem of solving for the function u, when

$$\int_0^x K(x,t)u(t)\,dt = g(x),\ 0 \le x \le T. \tag{1}$$

Here, the kernel K and the function u are assumed to be exact, but the right hand side g represents observations and T is a fixed number. Equation (1) is a Volterra integral equation of the first kind. We deal with the particular case where g represents a sparse set of measurements at the points $\{x_1, \ldots, x_N\}$ of a fixed grid. The grid-points $x_1 < \ldots < x_N$ are assumed to be known exactly and the measured values g_i are approximations of the functional values $g(x_i)$. Then Eq. (1) may be replaced by

$$\int_0^{x_i} K(x_i,t)u(t)\,dt = g_i,\ i = 1, \ldots, N. \tag{2}$$

This problem occurs in porous media physics, and deriving u involves the unstable process of inverting a linear smoothing operator. The operator for this particular problem has no bounded inverse.

We show that a regularized solution of the inverse problem is obtained by constraining the solution space to monotone and convex functions. These constraints, which satisfy certain physical constraints, may be expressed in the form of linear inequalities. We then arrive at a solution which approximates the measured values as closely as possible, where the deviation is defined by a suitable norm. We present numerical experiments which compare different discretization schemes, and results for real experimental data.