Abstracts of MMA2015, May 26–29, 2015, Sigulda, Latvia

ON UNIVERSAL CLASS OF PERIODIC ZETA-FUNCTIONS

MINDAUGAS STONCELIS AND DARIUS ŠIAUČIŪNAS

Šiauliai University

P. Višinskio street 19, Šiauliai LT-77156, Lithuania

E-mail: `mindaugas.stoncelis@su.lt, siauciunas@fm.su.lt`

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ be a periodic sequence of complex numbers with minimal period $q \in \mathbb{N}$. The periodic zeta-function $\zeta(s;\mathfrak{a})$, $s = \sigma + it$, is defined for $\sigma > 1$, by Dirichlet series $\zeta(s;\mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s}$, and can be analytically continued to the whole complex plane, except for a possible simple pole at the point $s = 1$.

Universality of the function $\zeta(s;\mathfrak{a})$ was began to study in [1] and [4]. The universality theorem for $\zeta(s;\mathfrak{a})$ with multiplicative sequence $\mathfrak{a}$ was proved in [3]. A new kind of universality of the function $\zeta(s;\mathfrak{a})$ with some restriction on compact sets was considered in [2].

In the report, we consider the case when q is a prime number and $a_q = \frac{1}{\varphi(q)} \sum_{l=1}^{q-1} a_l$, where $\varphi(q)$ is the Euler function. Let $\mathcal{K}$ be the class of compact subsets of the strip $D = \left\{ s \in \mathbb{C} : \frac{1}{2} < \sigma < 1 \right\}$ with connected complements, $H(K)$, $K \in \mathcal{K}$, denote the class of continuous functions on K which are analytic in the interior of K, and let $H_0(K)$, $K \in \mathcal{K}$, be the subclass of $H(K)$ of non-vanishing functions on K. We say that $\zeta(s;\mathfrak{a})$ is universal if the inequality

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{ \tau \in [0,T] : \sup_{s\in K} |\zeta(s+i\tau;\mathfrak{a}) - f(s)| < \varepsilon \right\} > 0$$

with every $\varepsilon > 0$ is satisfied for all $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. If this inequality holds with all $K \in \mathcal{K}$ and $f(s) \in H(K)$, then $\zeta(s;\mathfrak{a})$ is called strongly universal. Let χ denote a Dirichlet character modulo q, and $b(q,\chi) = \sum_{l=1}^{q-1} a_l \chi(l)$. Then we have the following statement.

THEOREM 1. *Suppose that the sequence $\mathfrak{a}$ satisfies at least one of hypotheses: i) $a_m \equiv c$, $m \in \mathbb{N}$; ii) a_m is a multiple of a Dirichlet character modulo q; iii) $q = 2$; iv) only one number $b(q,\chi) \neq 0$. Then the function $\zeta(s;\mathfrak{a})$ is universal.*

If at least two numbers $b(q,\chi) \neq 0$, then the function $\zeta(s;\mathfrak{a})$ is strongly universal.

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