

SOME INFERENTIAL ISSUES ON THE LOG–LINDLEY DISTRIBUTION

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The Log–Lindley distribution was introduced in Gómez-Deniz et al. [1] as a transformation of the generalized Lindley distribution defined in Zakerzadeh and Dolati [3]; even more precisely, it is an exponential transformation of the two-parameter Lindley distribution defined in Shanker et al. [2]. They gave some nice applications of this new model to insurance and inventory management. Since the Log–Lindley distribution has bounded support, it was also used as a regression model for bounded responses, providing an appealing alternative to the beta regression model. Although in [1] the authors started the study of this distribution, several important inference issues are still unsolved. This work studies some of them. Specifically, we deal with the following.

- (i) Parameter estimation. The Log–Lindley distribution depends on two parameters. Trying to find their maximum likelihood estimators by solving the likelihood equations is not advisable since it entails to find the roots of a polynomial of degree n , n being the sample size. We show that a reparametrization is useful to overcome this problem. The benefits of such reparametrization are numerically illustrated by Monte Carlo simulation studies.
- (ii) Data generation. We show that the reparametrization considered for parameter estimation has the variate generation property. Specifically, the quantile function can be expressed in closed form in terms of the negative branch of the Lambert W function.
- (iii) Regression model. We show that the reparametrization proposed in [1] is not useful if one want to use the Log–Lindley distribution as a regression model, since it entails certain restrictions on the parameter space. Such restrictions cannot be dropped for the resultant probability density function to be proper. By contrast, the reparametrization considered in the current work provides a convenient starting point for building a regression model where the parameters vary freely. An application to a real data set on cost effectiveness of risk management practices is provided.

REFERENCES

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