AN ALTERNATIVE APPROACH TO LOWER AND UPPER APPROXIMATION OPERATORS

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Let $R$ be a reflexive transitive relation on a set $X$, and $A$ be an $L$-subset of $X$, that is a mapping $A : X \rightarrow L$ where $L = (L, \leq, \land, \lor, *, \mapsto)$ is a cl-monoid [1]. Following the works of many authors (see e.g. [2], [3], [4]) by a lower and upper approximations of an $L$-set $A$ we call respectively

$$l(A)(x) = \inf_{x' \in X} (R(x, x') \mapsto A(x')) \forall A \in LX,$$

$$u(A)(x) = \sup_{x' \in X} (R(x, x') \ast A(x')) \forall A \in LX.$$

An important property of this approximation is that $l(A(x)) \leq A(x) \leq u(A(x)) \forall A \in LX$. In case $L$ is an MV-algebra [5], residuation $\mapsto$ satisfies a double negation property $(a \mapsto 0) \mapsto 0 = a \forall a \in L$ and hence $a \mapsto 0$ could be viewed as a complement of element $a$ [5]. This allows to get a ”good” relation between lower and upper approximation operators: $l(A(x)) \mapsto 0 = u(A \mapsto 0)(x)$. This property has a clear counterpart in case of topology, where $l$ and $u$ approximation operators could be realized as interior and closure operators respectively. This inspires us to define now operators $lu$ and $ul$ starting with $l$ and $u$ and setting

$$lu(A)(x) = (u(A \mapsto 0)(x)) \mapsto 0,$$

$$ul(A)(x) = (l(A \mapsto 0)(x)) \mapsto 0,$$

which we view as alternative approaches of lower and upper approximations. In case of $MV$-algebra they obviously coincide with $l$ and $u$ respectively.

We study properties of operators $lu$ and $ul$, and their relations with ”classical” operators $l$ and $u$. In particularly we show that the inequalities $l \leq lu$ and $u \leq ul$ are always true.

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REFERENCES